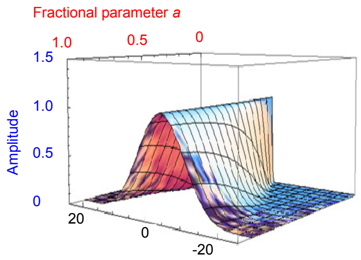
Opto-Electronic Engineering
光 电 工 程

Article
2018 , Vol 45 , Issue 6

DOI: 10.12086/oee.2018.170745

Analysis of PC6 window function using fractional Fourier transform



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Abstract: Fractional Fourier transform (FRFT) is a linear transform generalizing Fourier transform (FT) that plays an important role in the field of signal processing and analysis. FRFT contains an adjustable parameter α, which it rotates the signal in the time frequency plane and represents the signal in an intermediate domain between time and frequency. FRFT provides a measure about the angular distribution of signal's energy in time frequency plane. FT is a special case of FRFT when angle α is equal to $\pi/2$. This paper presents mathematical model for obtaining FRFT of PC6 window function. The different parameters of this window function are also obtained with the help of simulation results. A comparison of window function parameters is presented using FT and FRFT. Also comparison of this window function with Hanning window function is presented in terms of Side Lobe Fall off Rate (SLFOR). For different values of FRFT order, PC6 window function shows variation in different parameters. Thus by changing the FRFT order, the minimum stop band attenuation of the resulting window function can be controlled.

Keywords: fractional Fourier transform; Parzen window function; $\cos^6(\pi t)$ window function; PC6 window function; Hanning window function

Citation: Goel N, Kaur J. Analysis of PC6 window function using fractional Fourier transform[J]. *Opto-Electronic Engineering*, 2018, **45**(6): 170745

1 Introduction

Fourier transform (FT) is one of the most valuable tools in signal processing and analysis. Fractional Fourier transform (FRFT) is generalization to FT and belongs to class of time frequency representations [1-5]. FRFT is linear transform and depends upon an adjustable parameter α. It is called FT to the a^{th} power, where a needs not be an integer. Mathematically a^{th} order FRFT is the a^{th} power of FT operator so FRFT is also called fractional powers of ordinary FT. FRFT is effectively applied in all conditions where ordinary FT is used. FRFT is also called decomposition of the signal in terms of chirps. FRFT has found several applications in the areas of solution of differential equations [6], quantum mechanics [7], signal and image recovery [8], restoration and enhancement [9], space or time-variant filtering, multiplexing [10] and study of time frequency distributions [11-12].

Window functions have also played an important role in digital signal processing and analysis, digital filter design and speech processing [13-14]. These are used in harmonic analysis to reduce the effects of spectral leakage. A window function which gives a spectrum that has minimum leakage is preferred. In signal processing, a window function also known as an apodization function or tapering function is a mathematical function that is zero valued outside of some chosen interval. When a signal or function is multiplied by window function it also becomes zero outside the chosen interval [15]. A complete review of window functions and their properties using discrete FT (DFT) was presented by Harris [16]. Over time, many window functions [17-20], have derived to optimize some features of a new window and for easy implementation. A practical window usually has a trade-off between

Received 31 December 2017; accepted 21 February 2018

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the width of its main lobe and attenuation of its side lobes. No window function is found to be best in all aspects, so one should select a window function according to the requirement of a particular application [21].

Recently FRFT has been developed and utilized by number of researchers. FRFT is being used in almost all applications where FT is used. Analysis of Dirichlet and Generalized Hamming window functions has been carried out by Kumar et al. using FRFT in Ref. [22]. Also analysis of Kaiser and PC6 window function has been carried out using FRFT to show the dependence of main lobe width (MLW) on FRFT order and also an alternate methodology to tune the finite impulse response (FIR) filter transition band width (TBW) using FRFT [23]. In this paper mathematical analysis of PC6 window function along with its special cases has been carried out using FRFT. An attempt has also been made to study the variations of different window parameters such as half main lobe width (HMLW), maximum side lobe level (MSLL), -3 dB band width (-3 dB BW), -6 dB band width (-6 dB BW), SLFOR, coherent gain (CG) and equivalent noise band width (ENBW) [14, 16], by varying FRFT order. It is shown that for different values of FRFT order, this window function attain different values of these parameters. Also comparison of PC6 window function with Hanning window function is presented in terms of SLFOR.

The rest of paper is organized as follows. Section 2 gives an overview of FRFT. Section 3 describes PC6 window function. Results and discussion of window functions are presented in section 4 and finally the conclusion remarks are presented in section 5.

2 Fractional Fourier transform

FRFT is a very powerful tool in the field of signal processing for analyzing time varying signals. It is considered as generalization of ordinary FT with an additional parameter. It can be interpreted as rotation of signal by an arbitrary angle α in the time frequency plane. It represents the signal along the axis u making an angle α with the time axis. FRFT provides extra degree of freedom than FT as parameter α gives multidirectional application. The FRFT is a linear operator that corresponds to the rotation of the signal in time frequency plane through an angle which is not a multiple of π/2. FRFT is also a special case of linear canonical transform (LCT), which is a four parameter class of integral transform, and plays an important role in fields of optics and signal processing [24-26]. The continuous-time FRFT of a signal $x(t)$ is defined as [7]:

$$X_\alpha(u)=\int_{-\infty}^{\infty}x(t)K_\alpha(t,u)\mathrm{d}t \quad , \tag{1}$$

where $K_\alpha(t,u)$ is the kernel function and is given by

$$K_\alpha(t,u)=\begin{cases}\sqrt{\dfrac{1-\mathrm{i}\cot\alpha}{2\pi}}\cdot\mathrm{e}^{\mathrm{i}\left(\frac{u^2+t^2}{2}\cot\alpha-ut\csc\alpha\right)}, & \\ \qquad \text{if } \alpha \text{ is not multiple of } \pi & \\ \delta(t-u)\ , & \\ \qquad \text{if } \alpha \text{ is a multiple of } 2\pi & \\ \delta(t+u)\ , & \\ \qquad \text{if } \alpha+\pi \text{ is a multiple of } 2\pi & \end{cases} , \tag{2}$$

where the parameter α is called rotation angle of the transformed signal and is defined as $\alpha=a\pi/2$. However, it must be noted that 'a' needs not to be an integer. When $\alpha=\pi/2$, FRFT corresponds to FT, at $\alpha=0$, FRFT corresponds to identity operator, two successive rotations of signal through π/2 results in time inversion signal, three rotations of signal through π/2 results in inverse FT and four rotations leave the signal unaltered [27-28].

3 PC6 window function

PC6 window belongs to a special class of window functions designed for providing special characteristics like high SLFOR and far end attenuation. These windows are obtained by combining a lag window and data window in a linear manner. PC6 window is obtained by combining Parzen and $\cos^6(\pi t)$ window function. The amplitude of the side lobes in the frequency response can be decreased at the cost of increased HMLW for different values of window shape parameter [29-30]. When shape parameter equals to 1 or 0, Parzen and $\cos^6(\pi t)$ window function results. The PC6 window function in time domain is given by

$$w(t)=\begin{cases}\theta(l(t))+(1-\theta)d(t), & |t|\le 1/2\\ 0\,, & \text{elsewhere}\end{cases} , \tag{3}$$

where $0\le\theta\le 3.7$ and $l(t)$, $d(t)$ is given by

$$l(t)=\begin{cases}1-24|t|^2(1-2|t|), & |t|\le 1/4\\ 2(1-2|t|)^3\,, & 1/4\le|t|\le 1/2\end{cases} , \tag{4}$$

$$d(t)=\cos^6(\pi t)\,, \qquad |t|\le 1/2 \ . \tag{5}$$

Using equation (1) and (2), the FRFT of PC6 window function can be written as

$$\begin{aligned}W_\alpha(u)=&\sqrt{\frac{1-\mathrm{i}\cot\alpha}{2\pi}}\times\left\{\exp\left(\frac{\mathrm{i}u^2\cot\alpha}{2}\right)\right.\\ &\times\left[\theta\left(\int_{-1/4}^{1/4}\left(1-24|t|^2(1-2|t|)\right)\exp\left(\frac{\mathrm{i}t^2\cot\alpha}{2}-\mathrm{i}ut\csc\alpha\right)\mathrm{d}t\right.\right.\\ &+\int_{-1/2}^{-1/4}\left(2(1-2|t|)^3\right)\exp\left(\frac{\mathrm{i}t^2\cot\alpha}{2}-\mathrm{i}ut\csc\alpha\right)\mathrm{d}t\\ &\left.+\int_{1/4}^{1/2}\left(2(1-2|t|)^3\right)\exp\left(\frac{\mathrm{i}t^2\cot\alpha}{2}-\mathrm{i}ut\csc\alpha\right)\mathrm{d}t\right)\\ &\left.\left.+(1-\theta)\int_{-1/2}^{1/2}\cos^6(\pi t)\exp\left(\frac{\mathrm{i}t^2\cot\alpha}{2}-\mathrm{i}ut\csc\alpha\right)\mathrm{d}t\right]\right\}.\end{aligned} \tag{6}$$

Equation (6) can be rewritten as

$$W_\alpha(u)=\sqrt{\frac{1-\mathrm{i}\cot\alpha}{2\pi}}\times\exp\left(\frac{\mathrm{i}u^2\cot\alpha}{2}\right)\times\left[\theta(I_1+I_2+I_3)+(1-\theta)I_4\right]\quad,\tag{7}$$

here, we define:

$$E_1=\exp\left(\frac{\mathrm{i}u^2\cot\alpha}{2}\right),$$

$$E_2=\exp\left(\frac{\mathrm{i}t^2\cot\alpha}{2}-\mathrm{i}ut\csc\alpha\right),$$

I_1 and I_2 are given by

$$I_1=E_1\left[\underbrace{\int_{-1/4}^{1/4}E_2\mathrm{d}t}_{z_1}-48\underbrace{\int_0^{1/4}t^2E_2\mathrm{d}t}_{z_2}+48\left(-\underbrace{\int_{-1/4}^{0}t^3E_2\mathrm{d}t}_{z_3}+\underbrace{\int_0^{1/4}t^3E_2\mathrm{d}t}_{z_4}\right)\right]\quad,\tag{8}$$

$$I_2=E_1\left(2\underbrace{\int_{-1/2}^{-1/4}E_2\mathrm{d}t}_{z_5}+16\underbrace{\int_{-1/2}^{-1/4}t^3E_2\mathrm{d}t}_{z_6}+12\underbrace{\int_{-1/2}^{-1/4}tE_2\mathrm{d}t}_{z_7}+24\underbrace{\int_{-1/2}^{-1/4}t^2E_2\mathrm{d}t}_{z_8}\right)\quad,\tag{9}$$

I_3 and I_4 are given by

$$I_3=E_1\left(2\underbrace{\int_{1/4}^{1/2}E_2\mathrm{d}t}_{z_9}-16\underbrace{\int_{1/4}^{1/2}t^3E_2\mathrm{d}t}_{z_{10}}-12\underbrace{\int_{1/4}^{1/2}tE_2\mathrm{d}t}_{z_{11}}+24\underbrace{\int_{1/4}^{1/2}t^2E_2\mathrm{d}t}_{z_{12}}\right)\quad,\tag{10}$$

$$I_4=E_1\int_{-1/2}^{1/2}\cos^6(\pi t)E_2\mathrm{d}t\quad.\tag{11}$$

Equations (8), (9), (10) and (11) can be rewritten as

$$I_1=z_1-48z_2+48(z_3+z_4)\quad,\tag{12}$$

$$I_2=2z_5+16z_6+12z_7+24z_8\quad,\tag{13}$$

$$I_3=2z_9-16z_{10}-12z_{11}+24z_{12}\quad,\tag{14}$$

$$\begin{aligned}
I_4=&\frac{(-1)^{3/4}}{64\sqrt{\sin2\alpha}\csc\alpha}\cdot\sqrt{1+\frac{1}{-1+\exp(\mathrm{i}2\alpha)}}\\
&\cdot\exp\left(\frac{-\mathrm{i}\csc2\alpha(36\pi^2+u^2-u^2\cos2\alpha+24\pi u\sin2\alpha)}{2}\right)\\
&\cdot\{20\exp(\mathrm{i}6\pi\cdot\csc2\alpha+3\pi+2u\sin\alpha)\\
&\cdot\left[\mathrm{erfi}\left(\frac{(-1)^{1/4}(2u+\cos\alpha)}{2\sqrt{\sin2\alpha}}\right)-\mathrm{erfi}\left(\frac{(-1)^{1/4}(-2u+\cos\alpha)}{2\sqrt{\sin2\alpha}}\right)\right]\\
&+15\exp(\mathrm{i}2\pi\cdot\csc2\alpha\cdot(\pi(8+\cos2\alpha)+4\pi\cdot u\sin\alpha))\\
&\cdot\left[\mathrm{erfi}\left(\frac{(-1)^{1/4}(2u+\cos\alpha+4\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)-\mathrm{erfi}\left(\frac{(-1)^{1/4}(2u-\cos\alpha+4\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)\right]\\
&-15\exp(\mathrm{i}2\pi\cdot\csc2\alpha\cdot(\pi(8+\cos2\alpha)+8\pi\cdot u\sin\alpha))\\
&\cdot\left[-\mathrm{erfi}\left(\frac{(-1)^{1/4}(-2u+\cos\alpha+4\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)+\mathrm{erfi}\left(\frac{(-1)^{1/4}(-2u-\cos\alpha+4\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)\right]\\
&+6\exp(\mathrm{i}2\pi\cdot\csc2\alpha+5\pi+4\pi\cdot\cos2\alpha+2u\sin\alpha)\\
&\cdot\left[-\mathrm{erfi}\left(\frac{(-1)^{1/4}(2u+\cos\alpha+8\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)+\mathrm{erfi}\left(\frac{(-1)^{1/4}(2u-\cos\alpha+8\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)\right]\\
&+6\exp(\mathrm{i}2\pi\cdot\csc2\alpha+5\pi+4\pi\cdot\cos2\alpha+10u\sin\alpha)\\
&\cdot\left[\mathrm{erfi}\left(\frac{(-1)^{1/4}(-2u-\cos\alpha+8\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)-\mathrm{erfi}\left(\frac{(-1)^{1/4}(-2u+\cos\alpha+8\pi\sin\alpha)}{4\sqrt{\sin2\alpha}}\right)\right]\\
&+\exp(\mathrm{i}6\pi\cdot\csc2\alpha+3\pi\cdot\cos2\alpha+4u\sin\alpha)\\
&\cdot\left[\mathrm{erfi}\left(\frac{(-1)^{1/4}(12\pi\cdot\sin\alpha-2u-\cos\alpha)}{2\sqrt{\sin2\alpha}}\right)-\mathrm{erfi}\left(\frac{(-1)^{1/4}(12\pi\cdot\sin\alpha-2u+\cos\alpha)}{2\sqrt{\sin2\alpha}}\right)\right]\\
&+\exp(\mathrm{i}18\pi^2\cdot\cot2\alpha)\\
&\cdot\left[\mathrm{erfi}\left(\frac{(-1)^{1/4}(12\pi\cdot\sin\alpha+2u+\cos\alpha)}{4\sqrt{\sin2\alpha}}\right)-\mathrm{erfi}\left(\frac{(-1)^{1/4}(12\pi\cdot\sin\alpha+2u-\cos\alpha)}{4\sqrt{\sin2\alpha}}\right)\right]\}\quad.
\end{aligned}\tag{15}$$

Solving for I_1, I_2, I_3 and I_4, the FRFT of PC6 window function can be obtained as

$$\begin{aligned}
W_\alpha(u)=&\sqrt{\frac{1-\mathrm{i}\cot\alpha}{2\pi}}\\
&\cdot\{\theta[z_1-48z_2+48(z_3+z_4)\\
&+2z_5+16z_6+12z_7+24z_8\\
&+2z_9-16z_{10}-12z_{11}+24z_{12}]\\
&+(1-\theta)I_4\}\quad.
\end{aligned}\tag{16}$$

4 Results and discussion

The plots for calculating MSLL, HMLW, -3 dB BW, -6 dB BW and SLFOR of PC6 window function and special cases of this window function, which are Parzen and $\cos^6(\pi t)$ window function are shown in Figure 1. The comparison plots with Hanning window Function and continuum of these window functions to sinc pulse, as the FRFT order is varied is also shown in Figures 2 and 3.

The values of different parameters for PC6 window function are tabulated in Tables 1 to 3 respectively, for various values of FRFT order *a*. For Parzen window function, which is a special case of PC6 window function at θ equals to 1, as FRFT order is increased MSLL decreases from -51.41 to -53.00 dB, HMLW increases from 0.54 to 3.24 bins, -3 dB BW increases from 0.34 to 1.82 bins, -6 dB BW increases from 0.44 to 2.55 bins. SLFOR decreases to -24.00 dB, CG increases from 0.04 to 0.76 and ENBW decreases to 3.84 bins. Similarly for $\cos^6(\pi t)$ window function, which is also a special case of PC6 window function at θ equals to 0, when FRFT order is increased, MSLL decreases to -60.95 dB, HMLW increases to 3.91 bins, -3 dB BW increases from 0.38 to 2.24 bins, -6 dB BW increases to 3.08 bins. SLFOR first decreases to -48.32 and then increases to -42.00 dB, CG increases to 0.74 and ENBW decreases to 1.46 bins.

For PC6 window function θ is taken 1.29, and for this window function MSLL decreases to -50.03 dB, HMLW increases to 2.82 bins, -3 dB BW increases to 1.69 bins, -6 dB BW increases to 2.40 bins. SLFOR decreases to 24.05

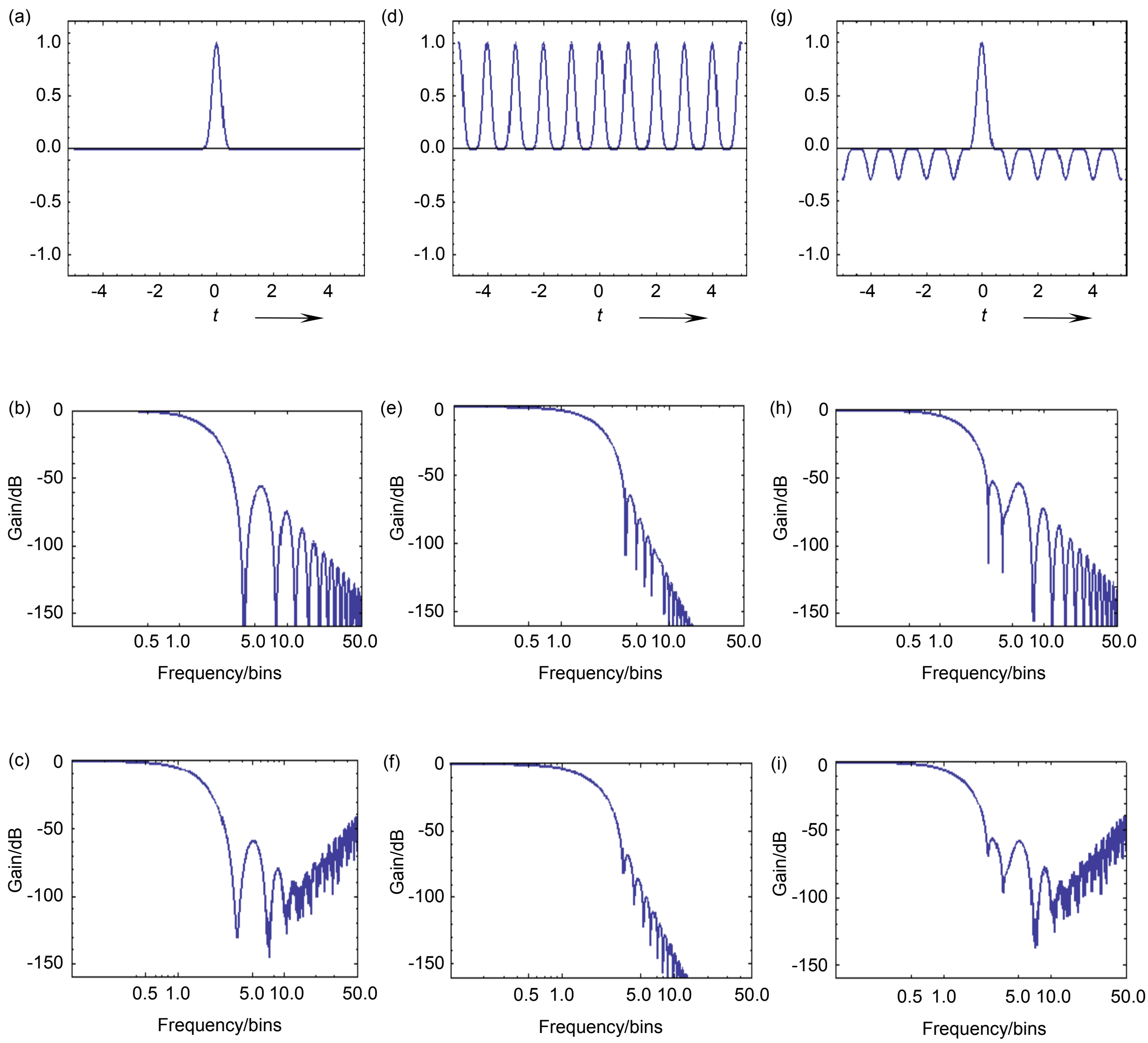

Fig. 1 Log magnitude plots of window functions. (a) Parzen window function in time domain; (b), (c) Log magnitude plot of Parzen window function at *a*=1 and 0.7; (d) $\cos^6(\pi t)$ window function in time domain; (e), (f) Log magnitude plot of $\cos^6(\pi t)$ window function at *a*=1 and 0.7; (g) PC6 window function in time domain; (h), (i) Log magnitude plot of PC6 window function at *a*=1 and 0.7

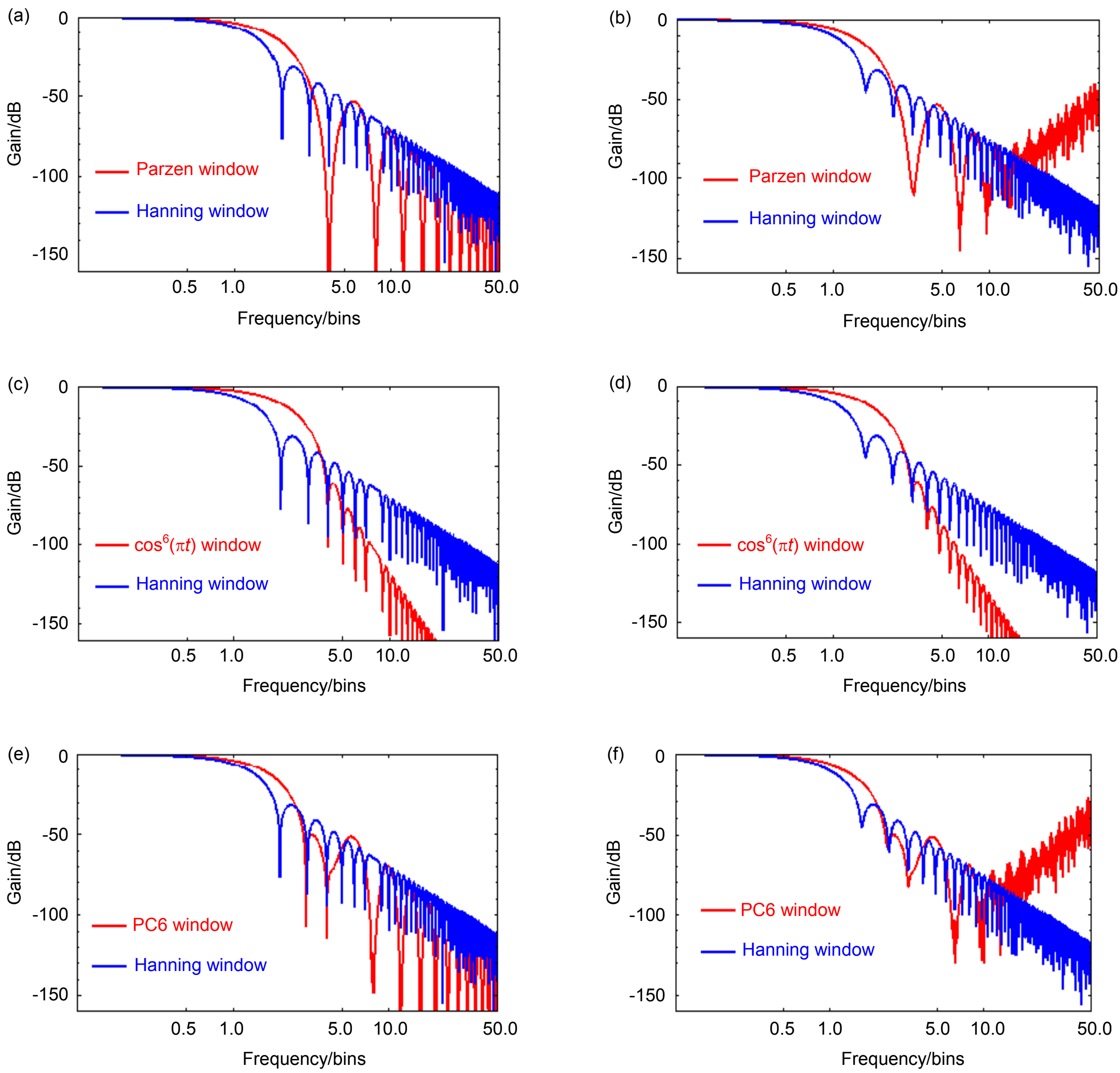

Fig. 2 (a), (b) Log magnitude plots of Parzen and Hanning window function at a=1 and 0.6; (c), (d) Log magnitude plots of $\cos^6(\pi t)$ and Hanning window function at a=1 and 0.6; (e), (f) Log magnitude plots of PC6 and Hanning window functions at a=1 and 0.6

dB, CG increases to 0.84 and ENBW decreases to 1.84 bins. Also comparison of PC6 window function is presented with Hanning window function as in Ref. [22] in terms of SLFOR. Also special cases of PC6 window function such as Parzen and $\cos^6(\pi t)$ are compared with Hanning window function. For a equals to 1, Parzen window function has SLFOR 24.00 dB, $\cos^6(\pi t)$ has 42.00 dB and PC6 has 24.05 dB but Hanning window function has 18.00 dB which is very low as compared to Combinational window function. And at 0.6 SLFOR for Parzen window is 31.29 dB, for $\cos^6(\pi t)$ is 48.33 dB and for PC6 window function SLFOR is 27.54 dB. But for Hanning window function, it is 18.20 dB and it has been found that SLFOR of PC6 window function is very high. Because of this high SLFOR this window function is used in applications such as FIR filter design and filter banks to attenuate far end attenuation. These window functions provide better far-end attenuation which suppresses the undesired interferences that occur in the filter bank. A window function with minimum stopband energy, better far-end attenuation and high SLFOR is the most suitable in such applications. Combinational window functions provide all these characteristics.

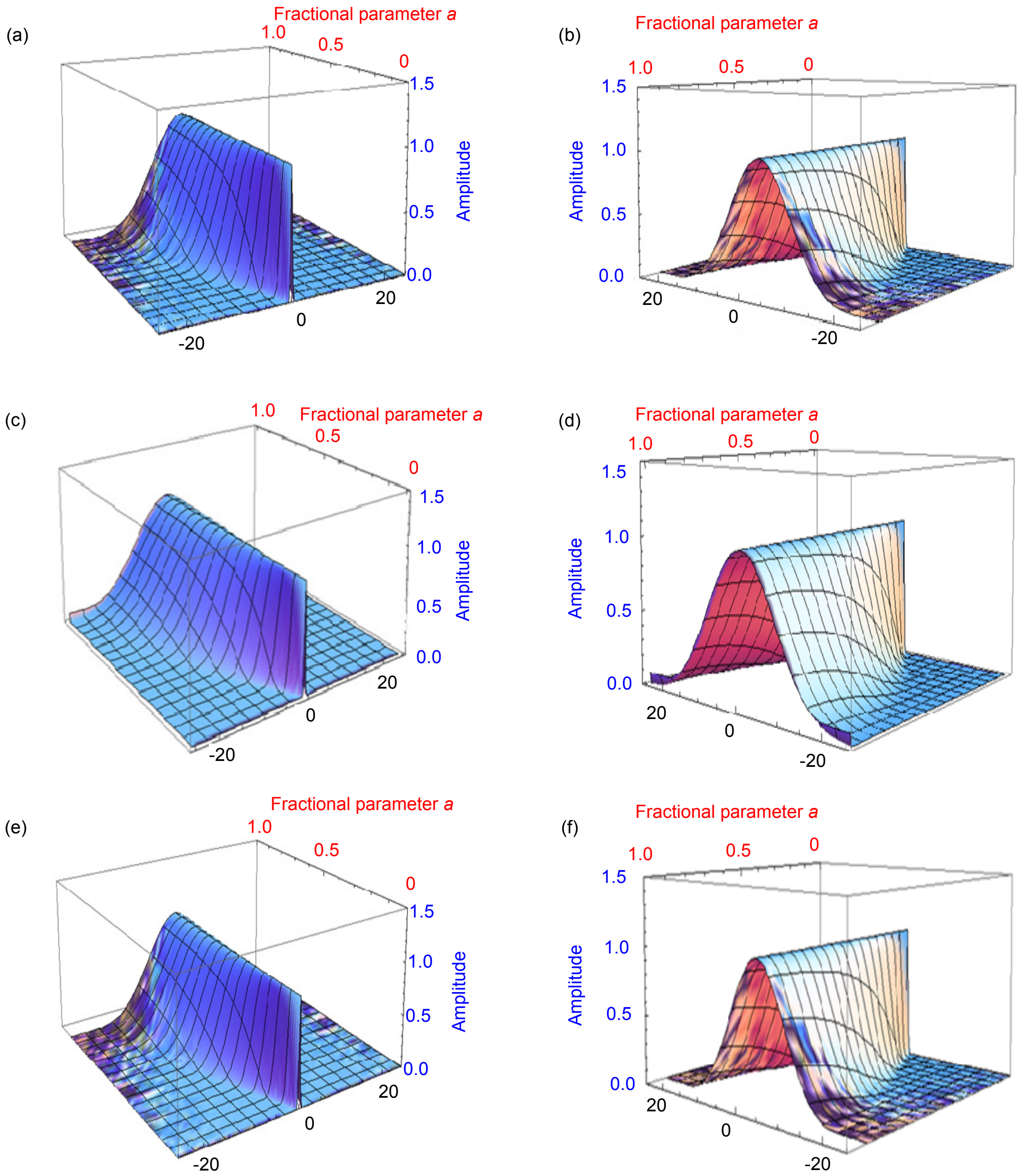

Fig. 3 (a), (b) The continuum of FRFT of Parzen window function; (c), (d) The continuum of FRFT of $\cos^6(\pi t)$; (e), (f) The continuum of FRFT of PC6 window function

5 Conclusions

The FRFT analysis of PC6 window function has carried out for different values of FRFT order *a*. For PC6 window function, when θ is 1, Parzen window results, and when θ is 0, $\cos^6(\pi t)$ window results and for PC6 window θ is taken 1.29. As FRFT order *a* is decreased, MSLL starts building up, HMLW, -3 dB BW and -6 dB BW are decreasing which is good for spectral analysis and SLFOR is decreasing. CG is decreasing and ENBW is increasing which is undesirable. The CG should be high and ENBW should be low to minimize spectral leakage.

Also comparison plots show that combinational window provide better far end attenuation as compared to Hanning window function. Thus this study reveals that there are variations in window parameters with variations in FRFT order. Using FRFT best solution can be obtained for variety of practical applications. So the choice of window function should be as per the concluded benefit and drawbacks to get optimum performance for specific applications.

Table 1 Parameters of Parzen window function (θ=1) with variations in FRFT order *a*

Sr. No.	*a*	MSLL/dB	HMLW/(bins)	-3 dB BW/(bins)	-6 dB BW/(bins)	SLFOR/ (dB/octave)	CG	ENBW/ (bins)
1	1.0	-53.00	3.24	1.82	2.55	-24.00	0.76	3.84
2	0.9	-52.89	3.20	1.82	2.52	-23.03	0.71	2.15
3	0.8	-52.79	3.08	1.80	2.44	-23.34	0.52	4.27
4	0.7	-52.72	2.88	1.68	2.28	-30.78	0.49	4.14
5	0.6	-52.67	2.64	1.48	2.08	-31.29	0.44	4.16
6	0.5	-52.79	2.30	1.34	1.82	-29.26	0.35	4.78
7	0.4	-52.76	1.91	1.10	1.50	-19.00	0.26	6.36
8	0.3	-52.52	1.49	0.86	1.18	-20.21	0.15	9.25
9	0.2	-52.48	1.02	0.59	0.80	-20.87	0.11	7.23
10	0.1	-51.41	0.54	0.34	0.44	-23.90	0.04	7.00

Table 2 Parameters of $\cos^6(\pi t)$ window function (θ=0) with variations in FRFT order *a*

Sr. No.	*a*	MSLL/dB	HMLW/(bins)	-3 dB BW/(bins)	-6 dB BW/(bins)	SLFOR/ (dB/octave)	CG	ENBW/ (bins)
1	1.0	-60.95	3.91	2.24	3.08	-42.00	0.74	1.46
2	0.9	-60.77	3.84	2.02	2.86	-48.32	0.29	16.10
3	0.8	-60.66	3.71	1.96	2.68	-48.44	0.27	16.75
4	0.7	-60.63	3.47	1.90	2.60	-48.65	0.26	16.50
5	0.6	-60.59	3.15	1.66	2.34	-48.33	0.24	15.00
6	0.5	-60.52	2.76	1.52	2.06	-48.14	0.20	15.05
7	0.4	-60.23	2.31	1.28	1.73	-48.05	0.14	20.33
8	0.3	-59.76	1.80	0.94	1.32	-47.68	0.12	19.02
9	0.2	-58.28	1.25	0.68	0.94	-46.58	0.12	6.26
10	0.1	-86.57	0.80	0.38	0.50	-46.57	0.06	5.46

Table 3 Parameters of PC6 window function (θ=1.29) with variations in FRFT order *a*

Sr. No.	*a*	MSLL/dB	HMLW/(bins)	-3 dB BW/(bins)	-6 dB BW/(bins)	SLFOR/ (dB/octave)	CG	ENBW/ (bins)
1	1.0	-50.03	2.82	1.69	2.40	24.05	0.84	1.84
2	0.9	-48.51	2.78	1.68	2.30	34.17	0.88	1.32
3	0.8	-49.94	2.69	1.60	2.16	26.38	0.54	3.93
4	0.7	-49.89	2.51	1.52	2.04	26.57	0.50	3.97
5	0.6	-49.83	2.30	1.40	1.98	27.54	0.44	4.00
6	0.5	-49.68	2.01	1.22	1.68	26.29	0.38	4.06
7	0.4	-49.54	1.68	0.98	1.36	26.32	0.27	5.11
8	0.3	-48.97	1.31	0.84	1.10	26.12	0.19	5.20
9	0.2	-47.14	0.92	0.56	0.76	25.43	0.12	5.65
10	0.1	-51.38	0.56	0.34	0.42	23.68	0.05	7.48

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Analysis of PC6 window function using fractional Fourier transform

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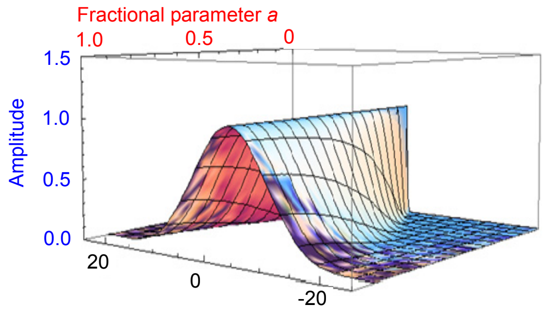

The continuum of FRFT of Parzen window function

Overview: Fractional Fourier transform (FRFT) is a linear transform generalizing Fourier transform (FT) that plays an important role in the field of signal processing and analysis. FRFT contains an adjustable parameter α, with which it rotates the signal in the time frequency plane and represents the signal in an intermediate domain between time and frequency. FRFT provides a measure about the angular distribution of signal's energy in time frequency plane. FT is a special case of FRFT when angle α is equal to $\pi/2$. Window functions have also played an important role in digital signal processing and analysis, digital filter design and speech processing. These are used in harmonic analysis to reduce the effects of spectral leakage. A window function which gives a spectrum that has minimum leakage is preferred. In signal processing, a window function also known as an apodization function or tapering function is a mathematical function that is zero valued outside of some chosen interval. When a signal or function is multiplied by window function it also becomes zero outside the chosen interval. A complete review of window functions and their properties using discrete FT (DFT) was presented by Harris. Over time, many window functions, have derived to optimize some features of a new window and for easy implementation. A practical window usually has a trade-off between the width of its main lobe and attenuation of its side lobes. No window function is found to be best in all aspects, so one should select a window function according to the requirement of a particular application.

This paper presents mathematical model for obtaining FRFT of PC6 window function. The different parameters of this window function are also obtained with the help of simulation results. A comparison of window function parameters is presented using FT and FRFT. Also comparison of this window function with Hanning window function is presented in terms of Side Lobe Fall off Rate (SLFOR). For different values of FRFT order, PC6 window function shows variation in different parameters. Thus by changing the FRFT order, the minimum stop band attenuation of the resulting window function can be controlled. The FRFT analysis of PC6 window function has been carried out for different values of FRFT order a. For PC6 window function, when θ is 1, Parzen window results, and when θ is 0, $\cos^6(\pi t)$ window results and for PC6 window θ is taken 1.29. As FRFT order a is decreased, MSLL starts building up, HMLW, -3 dB BW and -6 dB BW are decreasing which is good for spectral analysis and SLFOR is decreasing. CG is decreasing and ENBW is increasing which is undesirable. The CG should be high and ENBW should be low to minimize spectral leakage.

Also comparison plots show that combinational window provide better far end attenuation as compared to Hanning window function. Thus this study reveals that there are variations in window parameters with variations in FRFT order. Using FRFT best solution can be obtained for variety of practical applications. So the choice of window function should be as per the concluded benefit and drawbacks to get optimum performance for specific applications.

Citation: Goel N, Kaur J. Analysis of PC6 window function using fractional Fourier transform[J]. *Opto-Electronic Engineering*, 2018, **45**(6): 170745

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