Hartley transform for linear canonical transformation and uncertainty principle

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Hartley transform in LCT domain of $(1/2\pi)\exp(-x^2/2)$ with *b* is $1/2\pi$

Overview: Fourier transform is a basic tool in the field of signal processing, and with the in-depth research and the rapid development of the computer technology, researchers have managed more and more better results on signal processing. While more and more mathematical tools have been introduced into signal processing. Linear canonical transform is a generalization of Fourier transform and fractional Fourier transform. When researchers deal with the charp signal, they can obtain a very good effect by using the linear canonical transform. Based on the above reasons, more and more researchers begin to pay attention to linear canonical transform.

In this context, many transformations related to Fourier transform have been extended to fractional Fourier transform domains and linear canonical transform domains, such as the classical Wigner-ville distributions and cosine transformations. In Fourier transform domain, Hartley transform, which is the generalization of cosine transform, has a very significant advantage in the ability of transforming one real signal to another real signal, and it can delete the calculation of complex number hence it can cut down the calculation time. Because linear canonical transform kernel is complex to Fourier transform kernel, it is worthy to obtain Hartley transformation in linear canonical transform domain which transforms one real function to another real function. Based on the above issue, combined with linear canonical transform kernel, we define a Hartley transformation kernel, and then we obtain Hartley transformation in linear canonical transform domain. By simple calculations, Hartley transformation in linear canonical transform domain has two properties, which are transformed real function into real function and maintained parity invariant.

We know that the time resolution and the frequency resolution in the Fourier transform cannot be too small at the same time, which is the so-called Heisenberg uncertainty principle in Fourier transform domain. Based on the Heisenberg uncertainty principle in Fourier domain, one can also get the Heisenberg uncertainty principle for Hartley transform by some simple calculations. Since we have obtained the Hartley transformation in linear canonical transform domain, Hence, we guess that the Hartley transformation in linear canonical transform domain should also have the Heisenberg uncertainty principle. In this manuscripts, the Heisenberg uncertainty principle in linear canonical transform domain has been obtained for the real value function, while we simply discusses the entropy uncertainty principle of the Hartley transform domain.

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