Analog to information conversion for sparse signals band-limited in fractional Fourier transform domain

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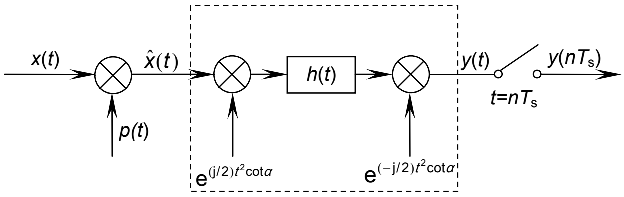

The structure of RD in fractional Fourier domain

Overview: The classical Shannon sampling theorem has a profound influence on signal processing and communication. For an analog signal $f(x)$ contains no frequencies higher than $W/2$ Hz, we can sample the signal uniformly at the rate of the W Hz, as prescribed by the theorem. Although the Shannon sampling theory is elegant and has proven to be fruitful, because of the constraints in hardware condition, it is difficult to sample radio frequency signals of very high bandwidth with analog-to-digital converter. As the development of sampling theorem, there are various extensions of the Shannon sampling theorem in the literature. Including sampling for functions of more than one variable, random processes, non-uniform sampling, generalized functions, and so on. In recent years, the Shannon sampling theorem has also been extended to the fractional Fourier transform (FRFT) which is a more general integral transform than the usual Fourier transform (FT).

The fractional sampling theorem is similar to the Shannon sampling theorem, which is concise and widely recognized. For an Ω_α band-limited signal whose fractional spectrum is at $[-u_m, u_m]$, we sample the signal at the fractional sampling rate u_s satisfied $u_s>2u_m$. Then we can realize the non-aliasing sampling and could represent exactly using the uniform samples of signal. However, the similar problem also exists just like Shannon sampling theorem, this fractional sampling theorem becomes impractical when the band limit of signal is very large because of the hardware cannot meet its demand.

With the increasing contradiction between high rate sampling and conversion accuracy, the traditional analog to digital conversion technology, which is based on the Shannon sampling theorem, is facing a great challenge, especially for the bottleneck effect on reducing the sampling rate. In recent years, the analog-to-information conversion (AIC) technology, which is based on the theory of compressive sensing, provides an effective method to solve this problem. However, the signal model of the existing AIC is only suitable for sparse signals band-limited in the Fourier transform (FT) domain. It cannot be applied to non-bandlimited chirp signals which are widely used in electronic information systems, including radar and communications.

Towards this end, we propose a new AIC based on the fractional Fourier transform (FRFT), which is not only the extension of the traditional AIC in the FRFT domain, but also can solve the problem as mentioned above. This novel sampling structure can greatly reduce the high sampling rate of signals, which has sparsity in fractional domain and especially for chirp signal. The theoretical derivation is presented, and the corresponding simulation analysis is also given. The simulation results are consistent with the theoretical analysis.

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