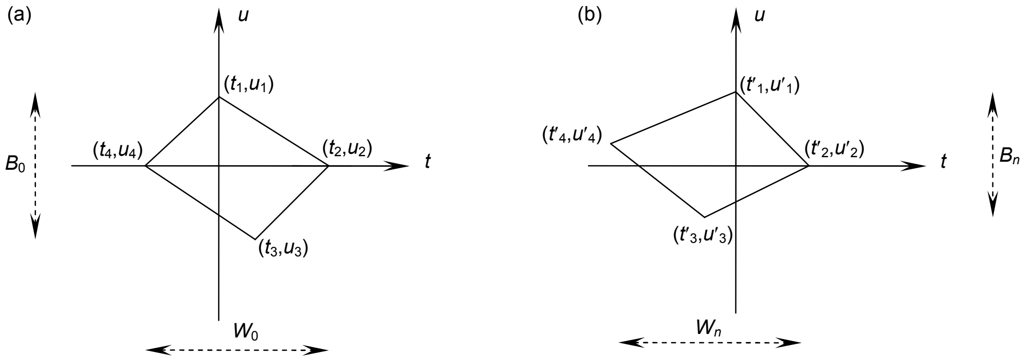
Research progress on discretization of linear canonical transform

Sun Yannan[1,2], Li Bingzhao[1,2*], Tao Ran[3]

[1]School of Mathematics and Statistics, Beijing Institute of Technology, Beijing 100081, China;
[2]Beijing Key Laboratory on MCAACI, Beijing 100081, China;
[3]School of Information and Electronics, Beijing Institute of Technology, Beijing 100081, China



Effect of LCT on arbitrary signal WVD. (a) Before LCT; (b) After LCT

Overview: Linear canonical transformation (LCT) is a generalized form of linear integral transform, which is a three-parameter linear integral transform. The LCT unifies a variety of transforms from the well-known Fourier transform (FT), fractional Fourier transform (FRFT) and Fresnel transform (also known as chirp convolution (CC)) to simple operations such as scaling and chirp multiplication (CM). The LCT is an important tool in optics because a broad class of optical systems including thin lenses, sections of free space in the Fresnel approximation, sections of quadratic graded-index media, and arbitrary concatenations of any number of these, sometimes referred to as first-order optical systems the paraxial light propagation can be modeled by the LCT. Besides, as a generalization of the transforms mentioned above, the LCT could be more useful and attractive in many signal processing applications, such as filter design, radar system, time-frequency analysis, phase reconstructions, and so on. On the other hand, the LCT can also be used in the fields of application mathematics, such as solution of differential equations. Therefore, the LCT has attracted a considerable amount of attention in many areas. In order to promote the applications of LCT, the discretization becomes the key vital issue of the LCT. Since the discretization of LCT cannot be obtained by directly sampling in time domain and LCT domain, it has been investigated recently. After the continuous LCT has been introduced, the definition and implementation of the discrete LCT (DLCT) have been widely considered by many researchers. Based on the development history of LCT discretization, a review of important research progress and current situation of discretization methods in the last nearly two decades is presented in this paper. The discretization algorithms include, discrete-time LCT, linear canonical series, discrete linear canonical transform. In this paper, the existing discretization methods are divided into three categories, directly discrete LCT, which appeared to have been first undertaken by Pei and Ding; operator decomposition LCT, which decomposed into products of these special operator combinations through the benefit of the additive property of the LCT; base decomposition fast discrete LCT, which was also first utilized by Hennelly and Sheridan to fast-implement DLCT; and other discrete LCT. Meanwhile, the connection among different discretization algorithms and the future development direction are given. It provides important reference value for researcher in the related fields, and can further promote its engineering application. With the deepening of research, LCT will be more and more widely used in practical applications.

Citation: Sun Y N, Li B Z, Tao R. Research progress on discretization of linear canonical transform[J]. *Opto-Electronic Engineering*, 2018, **45**(6): 170738

Supported by National Natural Science Foundation of China (61671063) and Foundation for Innovative Research Groups of the National Natural Science Foundation of China (61421001)
* E-mail: li_bingzhao@bit.edu.cn