Research progress in theories and applications of the fractional Fourier transform

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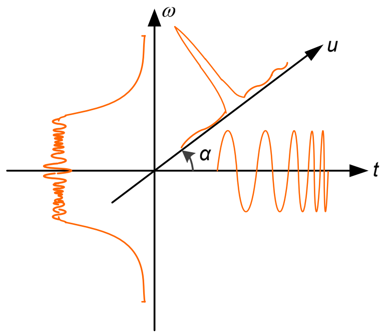

Chirp signal in time, frequency, and fractional domain

Overview: The fractional Fourier transform (FRFT) is a generalization of the Fourier transform. It has been received much attention since Namias provided its definition in the perspective of eigendecomposition and its application in quantum in 1980. FRFT can be interpreted as decomposition of a signal into chirp signals or rotation of the time-frequency plane with angle α. After years of research, the theoretical system of the FRFT has been relatively completed. Efficient and accurate discretization algorithms and sampling theorem associated with the FRFT make the digital signal processing based on discrete FRFT possible. Filtering and parameter estimation in fractional domains greatly promote applications of the FRFT in practice. Analysis of a signal in multiple fractional domains jointly distinguish signal processing utilizing FRFT from traditional signal processing, this is because with the rotation angle α changing from 0 to $\pi/2$, the FRFT of a signal can provide characteristics of the signal in many fractional domains, including time domain and frequency domain. Meanwhile, with the development of theoretical research, the FRFT also shows great values in practice. In addition to traditional areas such as quantum and optical, FRFT has also been applied in the area of signal processing, especially in radar signal processing, communication signal processing, image processing, medical signal processing, biology signal processing, and mechanical signal processing, et al. In this paper, we first provide definitions of the FRFT and its basic properties. We then review recent developments of the FRFT in theory, including discretization algorithms of the FRFT, various discrete fractional transforms derived from the discrete FRFT, sampling theory associated with the FRFT, filtering and parameter estimation in fractional domains, and joint analysis in multiple fractional domains. We next summarize progress in several application areas utilizing FRFT, including radar, communication, image encryption, optical measurement, health care, biology, and instrument. We also provide several future research directions of the FRFT, for example, fast algorithm and sparse sampling associated with the FRFT can be studied further to reduce complexity, existing applications of the FRFT can be promoted to improve the system performance further, FRFT can also be applied to machine learning because FRFT can provide characteristic of images in multiple fractional domains, FRFT based on graph may be very useful in graph signal processing, and discrete FRFT based on quantum computation may greatly reduce the complexity. By summarizing the research history, presenting research focus, and discussing future research directions of the FRFT, we try to provide a relatively comprehensive overview to the research progress in the FRFT to help readers to understand this filed better.

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