Athermal third harmonic generation in micro-ring resonators

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Introduction

Nonlinear high-harmonic generation in micro-resonators is a common technique used to extend the operating range of applications such as self-referencing systems and coherent communications in the visible region. However, the generated high-harmonic emissions are subject to a resonance shift with a change in temperature. We present a comprehensive study of the thermal behavior induced phase mismatch that shows this resonance shift can be compensated by a combination of the linear and nonlinear thermo-optics effects. Using this model, we predict and experimentally demonstrate visible third harmonic modes having temperature dependent wavelength shifts between \(-2.84 \text{ pm/}^\circ\text{C}\) and \(2.35 \text{ pm/}^\circ\text{C}\) when pumped at the L-band. Besides providing a new way to achieve athermal operation, this also allows one to measure the thermal coefficients and Q-factor of the visible modes. Through steady state analysis, we have also identified the existence of stable athermal third harmonic generation and experimentally demonstrated orthogonally pumped visible third harmonic modes with a temperature dependent wavelength shift of 0.05 pm/ºC over a temperature range of 12 ºC. Our findings promise a configurable and active temperature dependent wavelength shift compensation scheme for highly efficient and precise visible emission generation for potential 2f–3f self-referencing in metrology, biological and chemical sensing applications.

Keywords: third-harmonic generation; thermodynamics; micro-resonators

Kerr frequency comb generation\textsuperscript{17–26}, understanding and controlling the thermal behaviors of these modes remain a challenge in maximizing their THG efficiency\textsuperscript{27}, as both the linear and nonlinear thermo-optical (TO) effects impart refractive index variations of the waveguide\textsuperscript{19–26,28–42}. For platforms with a positive linear TO coefficients, the linear TO effect gives rise of tens of pm/°C temperature dependent wavelength shifts (TDWS) to the cold-cavity resonances of MRRs\textsuperscript{17–19}. It is possible to compensate the intrinsic linear TDWS by adding a cladding having a negative TO coefficient, such as polymer\textsuperscript{31–36} or TiO\textsubscript{2}\textsuperscript{37–38} and liquid crystal\textsuperscript{19} to offset the thermal dependency\textsuperscript{30}. These types of structures can achieve nearly athermal operation over temperature ranges of tens of degrees. However, the THG process must also account for the nonlinear TO effects as the localized heat generated by the high intra-cavity pump power can also introduce a redshift to the resonance\textsuperscript{9,28–29}. While this nonlinear TO effect can be compensated by the photorefractive effect in LN waveguide devices\textsuperscript{23–24} or by changing the structure of MRRs\textsuperscript{41} or by adopting the electrical or optical cooling approaches\textsuperscript{26,42}, these compensation methods are either unavailable or add complexity to the design in the complementary metal oxide semiconductor (CMOS) compatible MRRs devices.

Therefore, it is necessary to explore other approaches in the realization of temperature insensitive operation that will also address effects such as Kerr nonlinearity induced phase shift in the determination of the phase matching condition.

This work presents the development of a dynamic model to investigate the thermal dependency of the THG phase matching condition, which is affected by the linear and nonlinear TO effects induced phase mismatch. By using this model, we indirectly measure the thermal coefficients as well as Q-factor of the visible mode. We also perform the steady state analysis to investigate the TDWS of the TH modes as well as the existence of athermal THG operation on the thermal mismatch between the pump mode and the TH modes. We are able to verify the model’s predictions experimentally in a series of silicon rich HDSG add-drop MRR filters. We further classify the different types of visible TH modes from their THG efficiencies as well as TDWS. By precisely pairing the pump and the TH modes at different thermal mismatches, it is now possible to configure effective TDWS of the TH modes for a given MRR leading to athermal THG at different wavelengths.

Principle and theory

Steady-state thermal dynamics model

In this work, we consider both linear and nonlinear TO effects of THG in MRRs. The discussion requires steady and uniform intra-cavity power distribution for both the pump and the TH waves, in which both optical power and net heat are assumed to be evenly distributed over the ring cavity, so that the averaged local thermal nonlinearity still correlates with the Kerr nonlinearity in the moving reference frame. Experimentally, this uniform intra-cavity power distribution is achieved by injecting continuous wave (CW) laser into the resonance with large normal dispersion. Different from generating Kerr frequency combs\textsuperscript{17–19}, the sweep speed of the pump wavelength has to be slow so that the cavity can be thermally self-stabilized.

When intra-cavity power of the pump $I_p$ is much larger than $I_o$, the intra-cavity power of TH emission, it is reasonable to neglect the self-phase modulation (SPM), cross-phase modulation (XPM), and the pump depletion induced by the TH emission. By using the steady-state assumption for $I_p$ and $I_o$, from the coupled thermal dynamics model for THG in MRRs (See Supplementary Discussion III), we have\textsuperscript{11}

$$\begin{align*}
-i[\Omega_p - i\xi_p \omega_p \delta T - \frac{\kappa_p}{2}]a_p - i(\Theta_p + g_{pp}) I_p a_p - i\sqrt{\kappa_p} I_p = 0, \\
-i[\Omega_t - i\xi_t \omega_t \delta T - \frac{\kappa_t}{2}]a_t - i(\Theta_t + g_{tt}) I_p a_t = 0.
\end{align*}$$

(1)

where, $|a_p|^2 = I_p/h\omega_p$ and $|a_t|^2 = I_o/h\omega_o$ corresponds to the photon numbers of the pump and TH emission, respectively, and $\omega_p$ is the angular frequency of input pump. $\Omega_p = \omega_p - \omega_p$ and $\Omega_t = \omega_t - \omega_t$ are the cold-cavity resonance detuning of the pump and TH wave, respectively, where $\omega_p$ and $\omega_t$ are the cold-cavity resonance frequencies of pump and TH modes at temperature $T_o$, respectively. Here, $\delta T = T - T_o$ is the difference between temperature $T$ and $T_o$. $\xi_p$ and $\xi_t$ are the linear TO coefficients of the pump and TH modes, respectively\textsuperscript{29}, $\kappa_p$ and $\kappa_t$ are the overall losses of the pump and TH modes, respectively. Here, we define $\Theta_p$ and $\Theta_t$ as the nonlinear TO shift rate of the pump and the TH emission in rad/I, (See Supplementary Discussion III), respectively. $g_{pp}$ is the nonlinear factors of the pump and $g_{tt}$ is the XPM factors.
of TH emission and \( g_{\text{TH}} \) is the growth rate of the THG. In Eq. (1), \( k_{\text{ext}} \) is the external coupling rate of the pump and \( |\beta|^2 = P/\hbar\omega_0 \) refers to the photon number of \( P \). In Eq. (2), XPM effect dominates the Kerr nonlinearity.

**Tunable TO phase mismatch for THG in MRRs**

The overall THG efficiency depends strongly on the phase matching condition between the pump and the TH modes. It is more convenient to express the phase matching condition in terms of the phase mismatch \( \Delta\beta_{\text{total}} \) between these modes, which can be approximately expressed as

\[
\Delta\beta_{\text{total}} = \frac{\Delta\eta}{c} \left[ \Omega_{\text{TL}} + \Omega_{\text{TNL}} (\delta T) + \Omega_{\text{TNL}} (\Delta T) + \Omega_{\text{KNL}} \right],
\]

where, the terms \( \Omega_{\text{TL}} = \frac{\xi_1}{\omega_0^3} \delta T \), \( \Omega_{\text{TNL}} = \Theta_1 T_0 \) and \( \Omega_{\text{KNL}} = g_p^0 \) represent the linear TO phase mismatch, the nonlinear TO phase mismatch, and the Kerr nonlinear phase mismatch in THG, respectively. Here, \( \Delta\eta \) is the difference of group velocity and \( c \) is the speed of light in vacuum. From Eq. (2), we can analytically write the dependence of \( I_t \) of the TH mode as

\[
I_t = 12 \cdot \frac{g_p^2 I_p^3}{\kappa_t^1 + 2\xi_1 \omega_0^2 + \Omega_{\text{TL}} + \Omega_{\text{TNL}} + \Omega_{\text{KNL}}}. \tag{4}
\]

As shown in Eq. (4), the maximum efficiency occurs at its minimum mismatch. We also notice that, due to the nonlinear phase mismatches, the intensity of THG signal is proportional to the intra-cavity energy instead of the input pump power. Therefore, we should define \( \eta = 4 g_p^2 I_p^3 / (3\xi_1^2) \) as the overall THG efficiency in J⁻¹. For HDSG MRRs, since the calculated Kerr nonlinear coefficient of the TH emission \( g_{\text{TH}} = 1.35 \text{ W}^{-1}\text{m}^{-1} \) at 517 nm, we can neglect the \( \Omega_{\text{KNL}} \) term with a factor \( g_p^0 < 6.24 \times 10^7 \text{ pJ}^{-1} \) in Eq. (2) in the analysis which is two orders of magnitude smaller than \( \Theta_1 \), as shown in Table 1.

Figure 1(a) illustrates the individual contributions to the cavity resonance tuning due to the linear and nonlinear TO effects. It shows that while the \( \Omega_{\text{TNL}} \) can only produce a redshift, \( \Omega_{\text{TL}} \) can produce a red or blue shift depending on whether the temperature is ramped up or down [4-19]. From the nearly phase matching assumption, i.e. \( \Delta\beta_{\text{total}} \approx 0 \), we have \( \Omega_{\text{TL}} + \Omega_{\text{TNL}} = \xi_1 \omega_0^2 \delta T + \Theta_1 T_0 \approx -\Theta_1 \) from Eq. (3) which implies a decrease of \( \delta T \) resulting in a negative \( \Omega_{\text{TL}} \) that can be used to compensate the increase of \( \Omega_{\text{TNL}} \), as shown in Fig. 1(b). A better way to understand the interplay between the thermal behaviors of the pump and TH modes is to consider the thermal dynamic of

![Fig. 1](https://doi.org/10.29028/oea.2020.200028)

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these modes separately. We can denote $\tau_i = \xi_i \delta \omega_i / \Theta_i$ for $i = p$, $t$, as the ratio between the linear TO compensation rates of decreasing $\delta T$ and the nonlinear TO shift rate induced by $I_p$ for the pump and TH modes, respectively, as the measure of the effective TDWS of the modes. It is now possible to map the $\delta T - \lambda_p$ relationship and plot the TDWS of TH (red dot lines) and the pump mode (blue lines) separately as shown in Fig. 1(b), where $\lambda_p$ is the pump wavelength to obtain the resulting effective TDWS of the TH mode (red solid lines).

Figures 1(c)–1(e) show three scenarios of the effective TDWS of the TH mode in the $\delta T - \lambda_p$ diagrams at different values of $\tilde{\tau}$. As long as the TH mode is thermally matched with the pump mode, i.e. the thermal mismatch $\Delta \tau = \tau_p - \tau_t \approx 0$, their resonances can be kept correlated while maintaining the necessary phase matching condition, without the need for any external compensation, as shown in Fig. 1(d). In contrast, the thermal mismatch between the pump and TH modes will cause misalignment between their resonances and affect the THG efficiency. Figures 1(c) and 1(e) show when it is under- and over-compensated, respectively.

The existence of stable athermal THG in MRRs

In MRRs, the dependence of $I_p$ to the input pump power $P_0$ and the pump detuning $\Omega_p$ is a nonlinear relationship instead of the linear one\(^{44-45}\). In the HDSG MRRs, the dominant nonlinear TO effect further reduces the nonlinear threshold by two orders of magnitude. In this case, only configuring a fixed $\Delta \tau$ may not keep the thermal matching condition over a broad range of temperature. From Eq. (2), the existence of stable athermal modes can be found by equating $\partial \Omega / \partial \delta T = 0$. Applying this athermal condition into Eq. (1) leads to the following normalized equation which describes the dependence of $\Omega_p$ on $P$, and $I_p$ in nearly athermal THG via steady state analysis, (see Supplementary information Eq. S5.3).

$$F^2 = [1 + (\rho - \alpha)^2] \rho,$$

(5)

where, $\alpha = 2 \Omega_p / \kappa_p$ is the normalized pump detuning which is proportional to $\Omega_p$, $\rho = -2 \Theta_p \Delta \tau / \kappa_p$ is the normalized intra-cavity pump power, and $F^2 = 8 \Theta_p \kappa_p \Delta \tau P / (\kappa_p \tau_t)$ is the normalized intra-cavity pump power.

In Eq. (5), only when $|\alpha| > \sqrt{3}$, can there be three equilibria for one value of $F^4$, which is the criteria for the existence of stable athermal TH modes. Figure 2(a) plots the nonlinear relationship between $\alpha$ and $\rho$ using Eq. (5) with different $F^2$, in which the real roots of $\alpha$ are plotted as dashed lines. In Fig. 2(a), the green and blue solid lines show that $\alpha$ remains constant within a certain temperature range of $\delta T$, which implies the existence of stable athermal TH modes built up by fixed input pump powers. Moreover, different from the case of Kerr nonlinearity\(^{44-45}\), in athermal THG, the opposite sign of $\alpha$ implies that the dominating nonlinear TO effect can generate stable athermal modes on the blue edge of the pump resonance that are accessible by the sweeping of the pump signal.

![Fig. 2](image-url)
MRR fabrication and experimental setup

A series of HDSG four-port single ring MRR add-drop filters are fabricated for the investigation in which a ring resonator with radius of $R = 135 \mu m$ is side coupled to an input and output bus waveguide as shown in Fig. 2(b). The cross-section of both the rings and the bus waveguides are $2 \mu m \times 1 \mu m$, which consist of highly doped glass of refractive index 1.70 and cladded with silica. The Q-factors of the MRRs are varied by using a gap separation of 0.8 μm (R-1, R-3) and 1.0 μm (R-2), respectively. The corresponding transmission spectrum and the Q factors of the devices are shown in Figs. 1(a) and (b) as well as table 2 in ref. 13, respectively. The advantage of the devices are show n in Figs. 1(a) and (b) as well as table 2 in ref. 13, respectively. The advantage of the device having a drop port is to allow the direct monitoring of the TH mode emission at various $T$ of the different TM TH modes at two adjacent resonances of the MRR near 1550 nm is shown in Fig. 3(a). Fig. 3(b) shows the measured maximum intensities of the TH modes of Fig. 3(a) as a function of $I_p$. The result shows that it is feasible to classify these modes according to their THG efficiencies, with values in the order of $1.0 \, nJ^{-2}$, $0.05 \, nJ^{-2}$, $0.001 \, nJ^{-2}$, as type I, type II, and type III, respectively. Here, we calculated the THG efficiencies of type I TH modes by using $I_{\max} = R \xi \eta I_{\pmax} f(3 \xi^2) - R \eta I_{\pmax}^2$, with $\eta$ as the THG efficiency. Since we can only measure the scattered signal of the TH emission instead of directly measuring the intra-cavity TH emission energy, $\eta$ cannot be fully determined due to the unknown overall photodiode responsivity of the collimator $R$. The measured THG efficiencies $R \eta$ are 0.937 $nJ^{-2}$ in R-1 and 0.454 $nJ^{-2}$ in R-2.

Besides the THG efficiencies, each type has its own unique values of $\xi$, and $\Theta$, that can be extracted from the effective TDWS measurement at two or more resonance locations of the modes. The calculated $\xi$, and $\Theta$, of the type I and type II TH modes in table 1 show that the linear TO coefficient $\xi$ of these TH modes have similar values as the pump mode $\xi_p$ where all four TM-TM THG cases having negative thermal matchings, with $\Delta \tau < 0$. This is caused by their linear TO phase mismatches over-equalizing the nonlinear TO phase mismatches. For these TH modes, their TDWS rates $d$ are proportional to the thermal mismatch $-\Delta \tau$.

Demonstration of configurable TDWS and athermal THG in MRRs

For HDSG MRRs, the measured nonlinear TO shift rate $\Theta_p$ of the TM pump mode decreases more rapidly than the TE mode across the C- and L-bands, (see Supplementary Fig. S2(b). The result indicates different polarization combinations of the pump mode and the generated TH mode give rise to $\Delta \tau$ with different signs, which can lead to a positive, negative, or even zero TDWS. In MRRs R-1 and R-2, with 200–300 mW TM pumps, TE athermal TH modes with TDWS of $|\Delta \tau| \leq 0.62 \text{ pm/}^*C$ can be achieved as shown in Figs. 4(a) and 4(b). Figure 4(c) compares the measured photon count on the peak of...
Fig. 3. (a) Measured pump wavelength corresponding to the peak resonances of three types of TM TH modes. The calibrated cold-cavity resonance wavelengths were obtained by subtracted $\Omega_{NL}$ which are shown in plus center symbols. The calibrated TDWS of Type I (green lines) and Type II (purple lines) TH modes are also shown. (b) Measured third harmonic photon counts (THPC) on the resonance peaks of the TH modes as a function of $\Omega$ showing the cubic TH-pump relationship. The solid lines show the cubic TH-pump relationship for comparison. (c – d) The measured $I_p$ (upper) and filter response (lower) of corresponding type I TH modes as functions of $\Omega$. The calibrated cold-cavity resonance wavelengths of the resonance peak of TH modes at different $T$ are shown in open diamonds. The corresponding fitted TDWS of Type I (green lines) and Type II (purple lines) TH modes are also shown in plus center symbols. The calibrated TDWS of Type I (green lines) and Type II (purple lines) TH modes are also shown in plus center symbols.

Fig. 4. (a – b) Measured spectra of all the TE athermal TH modes in MRRs of R-1 (a) and R-2 (b) that are thermal matched, i.e. $\Delta \sigma = 0$. In (a) and (b), the measured wavelengths of the resonance peak of TH modes at different $T$ are shown in open diamonds. The corresponding fitted TDWS of TH modes are shown in grey dashed lines. (c) The measured photon count on the peak of different athermal TH modes as a function of $I_p$. The solid lines show the cubic TH-pump relationship for (a) – (b) as well as Figs. 5(c) and 5(e) for comparison. The data and fitting curves for R-1, R-2, R-3 are shown in blue, red, and green, respectively.

Table 1 | The corresponding parameters of Figs. 3(a) and 3(b). Detailed parameters of corresponding pump mode parameter measurement are discussed in Supplementary information.

<table>
<thead>
<tr>
<th>Device</th>
<th>$\lambda_0$ (nm)</th>
<th>$\xi_p$ (1°C)</th>
<th>$\Theta_\Omega$ @39 °C (rad/pJ)</th>
<th>$\gamma_p$ (pJ/°C)</th>
<th>TH mode T (°C)</th>
<th>$\Delta \Omega$ (rad/pJ)</th>
<th>$3\xi$ (pm/°C)</th>
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<tbody>
<tr>
<td>R-1</td>
<td>1548.88</td>
<td>1.23×10^{-5}</td>
<td>1.88×10^{3}</td>
<td>5.19</td>
<td>7.66×10^{-8}</td>
<td>6.84</td>
<td>Type I</td>
</tr>
<tr>
<td></td>
<td>1550.28</td>
<td>1.21×10^{-5}</td>
<td>1.62×10^{3}</td>
<td>5.19</td>
<td>7.66×10^{-8}</td>
<td>6.84</td>
<td></td>
</tr>
<tr>
<td>R-2</td>
<td>1549.16</td>
<td>1.29×10^{-5}</td>
<td>1.88×10^{3}</td>
<td>5.22</td>
<td>9.95×10^{-8}</td>
<td>6.89</td>
<td>Type I</td>
</tr>
<tr>
<td></td>
<td>1550.73</td>
<td>1.28×10^{-5}</td>
<td>1.80×10^{3}</td>
<td>5.40</td>
<td>7.69×10^{-8}</td>
<td>8.77</td>
<td>Type II</td>
</tr>
</tbody>
</table>

*Fitting data within $T = 31 °C \sim 45 °C$. **Fitting data within $T = 47 °C \sim 55 °C$. **Values cannot be extracted due to the measured wavelength shift is below the OSA resolution limit.
TH modes as a function of $I_p$. As shown in Fig. 4(c), in device R-2 the generated TE athermal TH mode has a THG efficiency of $R_{T2} = 0.0226\ \text{nJ}^{-1}$, which is one order of magnitude smaller than that of type I TM-TM THG combination. We also notice that the measured THG efficiency of the TE athermal modes is proportional to the Q-factor of MRR. The device R-2 with higher Q-factor can achieve THG efficiency one order of magnitude greater than that in R-1.

To verify the results, a second device R-3 is measured, having the same design as R-1. As shown in Fig. 4(c), the two measured TE athermal TH modes in R-3 have similar THG efficiencies as R-1, which indicates that they belong to one TH mode family. The measured spectra of these two athermal TH modes in MRR R-3 generated by TM-TE combination were shown in Fig. 5(c) and Fig.

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**Fig. 5** | (a) Thermal mismatch $\Delta \tau < 0$, with a TM pump generated a TM TH mode with a TDWS of $3d_{TM} = 7.05\ \text{pm/ºC}$. (b) Thermal mismatch $\Delta \tau > 0$, with a TE pump generated a TE TH mode with $3d_{TM} = -8.53\ \text{pm/ºC}$. (c) When thermal mismatch $\Delta \tau = 0$, with a TM pump generated TE TH mode with $3d_{TM} = 0.14\ \text{pm/ºC}$ (c) as well as $3d_{TM} = -0.27\ \text{pm/ºC}$ (f). (d, g) The extracted Q-factor of TE TH modes by using the data in (c) at 26 ºC and in (f) at 46 ºC, with Eq. (4) is used to generate the fitted lines. (e, h) The temperature dependence of the athermal TH mode resonance fluctuation with fixed $\Delta \tau$ and $F^2$ for the TH modes shown in (c) and (f), with the measured values in gray squares. In (a – c) and (f), the measured wavelengths of the resonance peak of TH modes at different $T$ are shown in open diamonds. The corresponding fitted TDWS of TH modes are shown in grey dashed lines.
5(f). In the same device, Fig. 5(a) shows that, with the TM pump mode, the generated TM TH visible mode yields $d_t = 2.35$ pm/°C, which is close to that of R-2 shown in Table 1. In this case, a negative $\Delta \tau$ leads to a positive TDWS. For the TE-TE combination in Fig. 5(b), it gives rise to a negative TDWS of $d_t = -2.84$ pm/°C due to the under-compensation of the linear TO effect. The results indicate that the TE-TE and TM-TM combination having opposite TDWS trends. Athermal operation with perfect thermal matching $\Delta \tau \approx 0$ can be achieved with the TM-TE combination in R-1 (Fig. 4(a)), R-2 (Fig. 4(b)), and R-3 (Figs. 5(c) and 5(f)). Figure 5(c) show an TDWS at the TH wavelength of only $|\Delta \rho| \leq 0.09$ pm/°C can be achieved in R-3, which is a clear demonstration of the use of linear TO phase mismatch to equalize the nonlinear TO phase mismatch when thermal matching occurs. This also shows that by selecting different combinations of the pump and TH modes, one can control the direction of the TDWS via configuring the thermal mismatch $\Delta \tau$ in these HDSG MRRs.

For athermal TH modes, since the measured pump wavelength slope is less than the resolution of the OSA of 1 pm, it is not possible to calculate $\xi_t$ and $\Theta_t$ accurately as in Fig. 3(a). From Eq. (2), the nearly phase matching assumption of the athermal modes can be used to determine the pump detuning $\Omega_p$ and $T_o$. Figures 5(c) and 5(f) indicates the two TE TH modes belong to the same type of TH mode in MRR R-3 and have similar $\xi_t$ in the L-band. By assuming that the ratio of $\xi_t$ to $\Theta_t$ in R-3 is the same as that in R-1 as shown in Table 1, we get a trial value of $\xi_t = 9.0 \times 10^{-6}$ /° C. From the data of Figs. 5(c) and 5(f), we can obtain $\Delta \tau = 0.36$ pJ/°C and $\Theta_t = 2.3 \times 10^9$ rad/pJ for Fig. 5(c) and $\Delta \tau = 0.29$ pJ/°C and $\Theta_t = 1.9 \times 10^9$ rad/pJ for Fig. 5(f) by using the estimated $\xi_t$.

One can also estimate $Q_{TH}$ of the TE TH mode by using the extracted $\Theta_t$ and $\xi_t$, along with the assumption of $\Delta \tau \approx 0$. Figs. 5(d) and 5(g) show the measured spectrum and fitted curve using Eq. (4) and the estimated values of $\Delta \tau$ and $\Theta_t$. As shown in Figs. 5(d) and 5(g), both of the athermal TH modes have a $Q_{TH}$ of $3.5 \times 10^5$, which is close to the intrinsic $Q$ of the TH mode and smaller than $Q_p = 4.32 \times 10^5$ of the pump mode.

### Discussion

For the HDSG MRR, the measured $\Theta_t$ is approximately two orders of magnitude higher than the shift rate from the Kerr effect. Such difference is about 10 times greater than that in the Si$_3$N$_4$ MRR. Meanwhile, both of these platforms have similar linear TDWS at around 20 pm/°C. It indicates that in the HDSG MRRs, the nonlinear TO effect can redshift the resonance of pump mode over a much broader bandwidth of about 10 times compared to the Si$_3$N$_4$ MRR. This covers more TH mode resonances and makes thermal matching easier to be achieved by finding a close to zero $\Delta \tau$ in the HDSG MRRs. We notice that MRRs with different gap separations between the bus and ring, coupling rates, and Q-factors, also lead to different wavelength dependent values of $\Theta_t$.

The model in Eq. (5) also provides insights to maximize the $\Theta_t$ value for broadening the temperature range of stable athermal THG. Since the real roots of $\alpha$ in Eq. (5) only exist when $|F| \geq |\rho|$ and the smaller roots cannot lead to an athermal mode, from $\alpha = \rho \pm \sqrt{F^2 / \rho - 1}$, we have

$$\Omega_p = \Theta_t \Delta \tau \delta T \left| Q_p \right| + \frac{\kappa_p^2 \rho^2 \left( \frac{k_p^2}{k_p^2 Q_p} \right)}{4}$$  \hspace{1cm} (6)$$

Equation (6) determines the dependence of athermal TH resonance frequency on temperature fluctuation $\delta T$, in which the first and second terms on the right-hand side of Eq. (6) represents the linear TO effect and the

### Table 2 | Comparison of athermal TDWS schemes using different micro resonators.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Temp. range (°C)</th>
<th>Effects to compensate</th>
<th>Compensation method</th>
<th>TDWS (pm/°C)</th>
<th>Gain</th>
<th>Polarization</th>
<th>Material</th>
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<tr>
<td>Ref.33</td>
<td>9–25</td>
<td>Linear TO</td>
<td>Polymer cladding</td>
<td>0.2</td>
<td>No</td>
<td>TE</td>
<td>SOI</td>
</tr>
<tr>
<td>Ref.35</td>
<td>25–90</td>
<td>Linear TO</td>
<td>sol-gel cladding</td>
<td>0.9</td>
<td>No</td>
<td>TM</td>
<td>SOI</td>
</tr>
<tr>
<td>Ref.37</td>
<td>22–28</td>
<td>Linear TO</td>
<td>a-TiO2 cladding</td>
<td>-1.6</td>
<td>No</td>
<td>-</td>
<td>Si</td>
</tr>
<tr>
<td>Ref.40</td>
<td>-</td>
<td>Nonlinear TO</td>
<td>w etched Si slab</td>
<td>N/A</td>
<td>No</td>
<td>TE</td>
<td>Si</td>
</tr>
<tr>
<td>This work</td>
<td>26–38</td>
<td>Linear TO &amp; nonlinear TO</td>
<td>Thermal matching</td>
<td>0.05</td>
<td>Yes</td>
<td>TE(TH) TM(pump)</td>
<td>HDSG</td>
</tr>
</tbody>
</table>
nonlinear TO effect, respectively. The increase of $|\delta\gamma|$ can simultaneously enhance the first term and reduce the second, and vice versa, which gives a similar form as the bandgap reference in electrical circuit design. With a non-zero and positive $\Delta r$, the linear TO effect can equalizes the nonlinear TO effect over a certain temperature range and provide temperature independent frequency reference when the pump frequency is fixed. In contrast to the external linear feedback control scheme, the thermal matching can automatically retain the phase matching of THG in the athermal mode.

Equation (6) also shows the operating temperature range of athermal THG can be determined by setting the calculated $\partial\Omega_{p}/\partial(\delta\Theta)$ to zero, which gives an approximately inverted cubic relationship between $\delta\Theta$ and $\tau$. Since the values of $\xi$ are on the same order of $\xi_{p}$, a close to zero $\tau$ can easily be obtained with a large $\Theta_{p}$ value, which is in the same order as $\Theta$ for broadening the operating temperature range. Contrarily, although the athermal THG still exists at small $\Theta_{p}$ values, it shrinks the operating temperature range to less than one degree and severely limits its effectiveness.

Using the parameters extracted from the measured athermal TH modes, the calculated $\alpha$ as functions of $\rho$ at a fixed $\Delta r$ are plotted in Figs. 5(e) and 5(h) using Eq. (5). The gray squares in Figs. 5(e) and 5(h) are from the measured result of Figs. 5(c) and 5(f), respectively. It shows a non-zero and positive $\Delta r$ (0.3 pJ/°C) can achieve perfect power insensitive athermal THG with less than hundreds of MHz variation within 12 °C and 6 °C ranges, respectively. This range is comparable to the reported passive athermal TDWS approaches for MRRs as shown in table 2. In Figs. 5(e) and 5(h), note that the variation of their wavelengths is only a few picometers, which is close to the accuracy limit of TLS and OSA.

Conclusions

In conclusion, we present a comprehensive study of the thermal behavior of the visible modes in microresonators generated by the THG process pumped with telecom wavelengths. A dynamic model that includes both the linear and nonlinear TO effects is used to explain the pump and third harmonic resonance shifts for a given input pump power in the MRRs. Using this model, we predict and experimentally demonstrate visible third harmonics modes having TDWS between −2.84 pm/°C and 2.35 pm/°C when pumped at the L-band. By precisely matching the pump and the TH modes at different thermal mismatches, it is now possible to configure effective TDWS for a given MRR leading to athermal THG at different wavelengths. We have also identified orthogonally pumped athermal visible TH modes with a TDWS of 0.05 pm/°C over a temperature range of 12 °C. In Table 2, we compared the proposed deterministic athermal THG scheme with other athermal approaches which use a negative TO coefficient overlay to compensate linear TO effect or equalize the nonlinear TO effect by optimizing the structure of MRRs. Unlike the previous works, our proposed scheme uses only one CW laser pump with fixed wavelength and power to directly generate athermal TH modes in a silicon rich CMOS-compatible MRRs without any external compensation scheme. This configurable THG approach can generate athermal visible emission mode with high Q-factor, approximately-zero TDWS, tens of degree temperature range, and no operation bandwidth limitation. This finding further promises the realization of temperature insensitive highly efficient THG for potential 2f−3f self-referencing in metrology, biological and chemical sensing applications.

References


Acknowledgements
We are grateful for financial supports from the Natural Science Foundation of Fujian Province (Grant No. 2017J01756); National Natural Science Foundation of China (Grant No. R-IND12101, No. 61675231); Strategic Priority Research Program of Chinese Academy of Sciences (Grant No. XDB24030300).

Author contributions
S. H. W. developed the original concept. B. E. L., R. R. D. and X. W. designed and fabricated the integrated devices. Y. L. performed the experiments. Y. L. and S. T. C. contributed to the development of the experiment. S. H. W., S. T. C., and B. E. L. contributed to the writing of the manuscript. L. W. and S. T. C. supervised the research.

Competing interests
The authors declare no competing financial interests

Supplementary information
Supplementary information for this paper is available at https://doi.org/10.29026/oea.2020.200028.