Perfect electromagnetic and sound absorption via subwavelength holes array

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Broadband sound absorption at low frequency is notoriously difficult because the thickness of the absorber should be proportional to the working wavelength. Here we report an acoustic metasurface absorber following the recent theory developed for electromagnetics. We first show that there is an intrinsic analogy between the impedance description of sound and electromagnetic metasurfaces. Subsequently, we demonstrated that the classic Salisbury and Jaumann absorbers can be realized for acoustic applications with the aid of micro-perforated plates. Finally, the concept of coherent perfect absorption is introduced to achieve ultrathin and ultra-broadband sound absorbers. We anticipate that the approach proposed here can provide helpful guidance for the design of future acoustic and electromagnetic devices.

Keywords: metasurface; absorber; acoustic; electromagnetics

Introductions

Perhaps all things around us are some types of waves, such as the light waves, acoustic waves, matter waves and even gravitational waves. The absorption or dissipation of the energy carried by these waves is of great importance in many applications ranging from energy harvest and information exchange to radar wave and sound noise control1–8. Among the various types of waves, the electromagnetic and acoustic waves share many similarities. In the last decades, the two disciplines have borrowed concepts from each other to grow up alternately. For example, the time-reversed sub-diffraction focusing technique was first developed by the acousticians and then utilized in the optical domain to break the diffraction limit9–12. On the other hand, the hyperlens was originally created in the optical domain and then introduced to acoustic research13,14. Recently, the concept of metasurface, an artificially structured thin film with property on demand, has become a hot spot in both the electromagnetic and acoustic regimes15–18. It is, however, surprising that many metasurface concepts such as Salisbury and Jaumann absorbers in the electromagnetic design have not been extended into the acoustic domain. Furthermore, although ultrathin and ultra-broadband electromagnetic absorbers have been realized19,20, similar thin absorbers are still difficult to obtain for acousticians4,17.

In this paper, we theoretically compared the boundary conditions for electromagnetic and acoustic metasurfaces. Similar to electromagnetic metasurface absorber, we show that various acoustic absorbers can be achieved using the effective impedance of deep-subwavelength perforated rigid plate (may be approximated by steel). In particular, an ultra-broadband and thin acoustic perfect absorber is demonstrated under coherent condition, showing excellent performance comparable to its electromagnetic counterpart.

Results

Analogy between the boundary conditions of electromagnetic and acoustic waves

First of all, the boundary conditions for electromagnetic metasurface are considered. As shown in Fig. 1(a), the tangential components of the electric and magnetic fields are continuous at the interface between two half-infinite materials. When a non-magnetic impedance sheet with zero thickness is placed at this interface21, the boundary conditions should be rewritten as
where $i$, $r$ and $t$ denote the incidence, reflection and transmission, $\| \|$ represents the tangential components, $\hat{n}$ is the unit vector along the normal direction, $Y_{\text{em},t}$ is the electric sheet inductance of the metasurface. In general, the effective impedance of a micro-tube perforated in a rigid plate should be calculated using computational fluid dynamics (CFD)\(^{23}\). However, when the tube is short compared to the wavelength, the effective impedance can be approximately analyzed using the equation of aerial motion

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial x^2} = -\eta \frac{\partial v}{\partial t},$$

where $v$ is particle velocity, $c$ is the velocity of acoustic wave. Combing equations (4)–(7), one can obtain the following expression

$$Z_{\text{ac},t} = Z_{\text{ac},s} + Z_{\text{ac},e} + Z_{\text{ac},t}.$$

Interestingly, equations (3) and (8) have very similar form, except that the admittance in equation (3) is replaced by the impedance in equation (8). This similarity implies that both the acoustic and electromagnetic problems in metasurfaces can be solved by transfer matrix, along with proper boundary conditions.

### Impedance sheet realized by micro-perforated plate

In the above discussion, an arbitrary sheet impedance is assumed without considering its practical realization. In the following, the effective impedance of micro-perforated plate (MPP) is utilized in the design of various acoustic absorbers. It should be noted that although the MPP absorbers have been intensively studied after its invention by Maa in 1970s\(^{22–24}\), its intrinsic similarity with electromagnetic absorber such as Salisbury and Jaumann absorbers has seldom been studied\(^{25–27}\). In this paper, we show that the concept borrowed from electromagnetics is very efficient in the design of acoustic devices.

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should be considered, leading to the following sheet impedance

\[ Z_s = \frac{t + \beta d}{t} Z_{s,\text{inc}}, \]  

(12)

where \( \beta = 0.613 \) is an empirical parameter\(^{23} \).

The normalized impedance at \( f=10 \) kHz for different hole diameters is plotted in Fig. 2. Clearly, as the increase of the hole diameter, the impedance drops down rapidly. When the hole is extremely small, the impedance tends to be infinite, with all sound wave being reflected. The frequency-dependent impedance of a MPP metasurface is shown in Fig. 3, where the geometric parameters are fixed as \( d=38.5 \) μm and \( b=t=100 \) μm. Evidently, the resistance is almost independent of the frequency, while the reactance is proportional to the frequency.

Fig. 2 | Normalized impedance versus the hole diameter \( d \) at a frequency of 10 kHz. \( b=t=100 \) μm.

Fig. 3 | Normalized impedance of micro-perforated plate. \( d=38.5 \) μm, \( b=t=100 \) μm.

From above results, it is clear that a thin micro-perforated plate bears similar properties with the resistive sheet (thin metal film) in the electromagnetic applications\(^{25} \). Combining with the similarity in boundary conditions, it can be concluded that the acoustic waves can be controlled using electromagnetic-inspired approaches. Furthermore, it is expected that more complex sheet impedance is possible by combing the micro-perforated plate with cavities or other resonant structures\(^{28} \).

Acoustic Salisbury and Jaumann absorbers

As the first step to demonstrate the intrinsic connection between the electromagnetic and acoustic metasurface, the acoustic-type Salisbury\(^{25,29} \) and Jaumann\(^{30} \) absorbers are investigated. As depicted in Fig. 4, the absorption spectra of two optimized absorbers are calculated using transfer matrix method (see methods). Theoretically, the physical mechanism behind the absorption is the multiple interference between successive reflections\(^{31} \). Although only theoretical results are given here, previous experiments agree well with our theory\(^{24} \), indicating that the following discussion is sound and robust.

For the Salisbury-type absorber, there is only one micro-perforated plate (labelled as MPP-1) separated from the rigid body with a distance of \( h_1=48 \) mm. The geometric parameters of MPP-1 is \( d_1=38.5 \) μm and \( b=t=100 \) μm. For the Jaumann-type absorber, there are four MPPs placed successively with distance \( h_1=h_2=h_3=h_4=12 \) mm. The thickness \( t \) and period \( b \) are equal for all these MPPs, while the hole diameters are \( d_1=38.5 \) μm, \( d_2=40 \) μm, \( d_3=45 \) μm and \( d_4=50 \) μm. Note that the one-layer Salisbury and four-layer Jaumann structures are two representing examples. For the two- and three-layer structures, the bandwidth would be larger than the one-layer but smaller than the four-layer cases. Similar to the electromagnetic case, the thickness mainly determines the absorption frequency. As the thickness becomes larger, the frequency will become smaller.

As illustrated in Fig. 5, the dependence of the acoustic Jaumann absorber on the incidence angle \( \theta \) is also investigated. Similar to its electromagnetic counterpart, the absorption efficiency would decrease as the rise of the
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incidence angle although most is larger than 90% for θ < 60°. In addition, we noted that the angle dependence is similar to the transverse electric (TE) mode, rather than the transverse magnetic (TM) mode, as can be derived from the boundary conditions.

Anyway, the results shown in Figs. 4 and 5 resemble that for classic electromagnetic absorbers. Consequently, we expect that the theory of the recent developed electromagnetic absorbers can be leveraged to design acoustic absorbers in a more sophisticated way.

Coherent perfect absorption of acoustic wave

Different from traditional absorbers, coherent perfect absorption (CPA) is a process that requires two or more coherent inputs. Owing to the revision of the operation condition, previous limitation on the bandwidth and absorber thickness, as set by the casualty and Kramers-Kronig relations, can be eliminated. This broadband coherent absorption characteristic seems a surprise since CPA was thought as a time-reversed process of lasing, which is intrinsically narrowband. Nevertheless, from the view point of electrodynamics, this broadband absorption can be indeed treated as the time-reversed process of the radiating current sheet, which is not limited in bandwidth. Although CPA is of particular importance for electromagnetic applications, its realization in acoustic region is hampered by the absence of proper lossy materials. Although there are some attempts to design acoustic CPA, the results seems not so satisfactory because only narrowband absorption can be achieved.

Figure 6 illustrated the absorption spectrum under CPA condition. The hole diameter of the MPP is 32.1 μm, while the thickness and period are both 100 μm. As we expected, the absorption can be as high as 99% at frequencies below 10 kHz, which is very similar to the ultra-broadband electromagnetic CPA. It should be noted that the normalized impedance for the MPP at CPA condition is nearly twice that of air, while in the electromagnetic case the impedance is only half. This difference is a direct result of the different definition of impedance in acoustics and electromagnetics.

One of the intrinsic advantages of CPA over traditional absorbers lies in the fact that the absorption can be easily tuned by varying the phase difference between the left and right inputs (l). Similar to the electromagnetic counterpart, the absorption may be written as:

$$A = 1 - \sin^2\left(\frac{\omega l}{2v_0}\right),$$

where $v_0 = 343$ m/s is the velocity of sound in air.

As shown in Fig. 7(a), when the path difference is fixed to be 0.2 m, the absorption spectra vary rapidly with the sound frequency. Note that the absorption at very low frequency is always near 100%, implying that this technique is very efficient for the absorption at low frequency. Figure 7(b) depicts the absorption at 10 kHz for various path difference, where the ratio of the minimum to maximum can be as smaller as 2e-5, which is beneficial to the coherent acoustic modulators. Similar to the concept of “controlling light with light”, the acoustic CPA can be also leveraged to realize “controlling sound with sound” on metasurfaces.
Discussions
In summary, we have unified the theory of electromagnetic and acoustic absorbers in the context of metasurfaces. It is demonstrated unambiguously that the theory of electromagnetic absorbers can provide useful guidance for acoustic researches. The ultra-broadband acoustic CPA proves that previous judgment about sound absorption, especially at low frequency, is not accurate\(^1\).

It should be pointed out that the impedance of the MPP is mainly determined by the radius of the deep-subwavelength-scale holes, which are also of particular importance in electromagnetic scenarios such as the extraordinary optical transmission\(^2\) and frequency selective surfaces\(^3,4\). As the operational frequency rises, the holes need to be scaled down to maintain the required impedance and there will be huge challenges to fabricate such small holes in a large area. Meanwhile, the accuracy of Maa’s model needs further experimental investigation at higher frequency. Nevertheless, there is no problem if we focus on the low-frequency regime, which is just the difficult point of traditional approaches. Potential applications may be found in the noise control in both air and water environments.

Method of transfer matrix for the multilayered acoustic absorber
Similar to the electromagnetic case, the multilayered acoustic structure can be calculated efficiently using the transfer matrix. As depicted in Fig. 8, the coefficients of the counter-propagating acoustic waves are denoted as \(A\) and \(B\), while the two pairs of quantities can be written as

\[
\begin{align*}
A_{m+1} &= \frac{1}{2Z_{m+1}} \left[ Z_{m+1} + Z_{m} + Z_{s,m} - Z_{m} + Z_{s,m} \right] \\
&\cdot \left[ \exp(-ikh_{m}) \quad 0 \right] \left[ A_{m} \right], \\
B_{m+1} &= \frac{1}{2Z_{m+1}} \left[ Z_{m+1} - Z_{m} - Z_{s,m} - Z_{m} + Z_{s,m} \right] \\
&\cdot \left[ 0 \quad \exp(ikh_{m}) \right] \left[ B_{m} \right],
\end{align*}
\]

where

\[
M_{1,m} = \frac{1}{2Z_{m+1}} \left[ Z_{m+1} + Z_{m} + Z_{s,m} - Z_{m} + Z_{s,m} \right] \\
\cdot \left[ \exp(-ikh_{m}) \quad 0 \right] \left[ A_{m} \right],
\]

is the transition matrix of the metasurface, while

\[
M_{2,m} = \begin{bmatrix}
\exp(-ikh_{m}) & 0 \\
0 & \exp(ikh_{m})
\end{bmatrix}
\]

is the propagation matrix of the \(m\)th layer. Here \(k = \cos(\theta)\omega / c\) is the normal component of the acoustic wave number.

Obviously, the amplitudes of acoustic waves at the outmost layer can be calculated from the innermost directly. Since there is a rigid body acting as a perfect reflector, the final form of the transfer matrix can be written as

\[
\begin{align*}
\begin{bmatrix}
A_{N+1} \\
B_{N+1}
\end{bmatrix} &= M_{N}M_{N-1} \cdots M_{2}M_{1} \begin{bmatrix} 1 \\ -1 \end{bmatrix},
\end{align*}
\]

where \(M_{m}=M_{m,\text{air}}M_{m}\) is the transfer matrix of a pair of air and metasurface. The reflection and absorption coefficients can be then calculated as

\[
\begin{align*}
\text{Reflection} &= \frac{B_{N+1}}{A_{N+1}}, \\
\text{Absorption} &= 1 - \left( \frac{B_{N+1}}{A_{N+1}} \right)^2.
\end{align*}
\]

References

Fig. 7 | Variation in the absorption under different coherent conditions. (a) Absorption versus frequency \(f\) with path difference of \(l=0.2\) m. (b) Absorption versus \(l\) at \(f=10\) kHz.
Fig. 8 | Schematic of the acoustic multilayer. In each layer, there are two counter-propagating waves with amplitudes of \(A\) and \(B\). These amplitudes are connected with matrix shown in the text.