Fiber-based mode converter for generating optical vortex beams

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In this work, an all-fiber-based mode converter for generating orbital angular momentum (OAM) beams is proposed and numerically investigated. Its structure is constructed by cascading a mode selective coupler (MSC) and an inner elliptical cladding fiber (IECF). OAM modes refer to a combination of two orthogonal \(L_{Plm}\) modes with a phase difference of \(\pm \pi/2\). By adjusting the parameters and controlling the splicing angle of MSC and IECF appropriately, higher-order OAM modes with topological charges of \(l = \pm 1, \pm 2, \pm 3\) can be obtained with the injection of the fundamental mode \(LP_{01}\), resulting in a mode-conversion efficiency of almost 100%. This achievement may pave the way towards the realization of a compact, all-fiber, and high-efficiency device for increasing the transmission capacity and spectral efficiency in optical communication systems with OAM mode multiplexing.

Keywords: optical vortices; fiber optics; singular optics; mode-division multiplexing


Introduction

Optical vortex beams (OVBs) possessing orbital angular momentum (OAM), i.e., OAM beams or OAM modes, with a helical phase wavefront indicated by the term \(\exp(il\phi)\), where \(\phi\) denotes the azimuth angle and \(l\) is an integer representing topological charges (TCs), have a phase singularity in the transverse phase center and a doughnut-shaped profile in the transverse intensity distribution.

In recent decades, mode-division multiplexing has increasingly attracted attention as a method for increasing the transmission capacity and spectral efficiency of optical communication systems, with a special focus on multiplexing technology with OAM modes because of their infinite value of \(l\) per photon\(^{2–4}\). In addition, OAM beams have been developed for applications such as optical tweezers and atom manipulation\(^{5–7}\), microscopy\(^8\), as well as quantum information processing\(^9\).

A pivotal challenge is the generation of OAM beams, for which various methods have been developed. Free-space optical systems, such as spiral phase plates\(^{10}\), cylindrical lens converters\(^{11}\), and spatial light modulators (SLM)\(^7–9\), are relatively bulky or expensive. Hence, various fiber-based generation schemes have been investigated, such as acoustic-optic interaction\(^{12}\), mechanical stress-induced long-period grating\(^{13,14}\), CO\(_2\) laser-inscribed long-period fiber grating\(^{15}\), offset splicing\(^{16}\) or pressure exertion\(^{17}\) in a single-ring fiber, and the superposition of two orthogonal \(LP_{11}\) modes with a micro phase difference, because of their superior robustness, low cost, compatibility, and high efficiency. In these previous papers, however, only OAM modes for \(l = \pm 1\) have been achieved, which makes the development of high-capacity all-fiber communication systems based on OAM modes difficult. Therefore, it is highly desirable to generate high-order OAM (HO-OAM) modes. In our previous work, we theoretically studied the generation of first-order OAM modes\(^{18}\) and HO-OAM modes by using a helical fiber Bragg grating\(^{19}\). Moreover, Fang et al. simulated the flexible generation of HO-OAM modes by using helical long-period fiber gratings\(^{20–21}\). Furthermore, theo-
retical methods for generating HO-OAM modes were proposed by phase-matching coupling between different vector modes. In addition, integrated photonics-based methods have been implemented for OAM multiplexing and generation, which realize relatively compact systems.

In this letter, an all-fiber-based mode converter for the efficient generation of OAM modes with TCs of \( l = \pm 1, \pm 2, \pm 3 \) is proposed and simulated in detail. The mode converter is developed by combining a mode-selective coupler (MSC) and an inner elliptical cladding fiber (IECF). The MSC is used to generate \( LP_{\text{in}} \) modes, where \( l \) denotes the azimuthal index and \( m \) represents the radial index, which is assumed as 1, and the IECF can introduce a phase difference of \( \pm \pi/2 \) between two degenerate orthogonal \( LP_{\text{in}}^0 \) and \( LP_{\text{in}}^1 \) modes after a certain propagation length. Consequently, the HO-OAM modes can be obtained with a high mode-conversion efficiency of close to 100%.

Mode-selective coupler

As a part of the proposed mode converter, the MSC is composed of a single-mode fiber (SMF) and a multimode fiber (MMF), which has been exploited to excite the high-order modes (HOM) \( LP_{\text{in}} \) in MMF with high efficiency. The MSC has been used to generate OAM beams. If the refractive-index difference between the core and cladding is small enough to satisfy the weak guiding approximation, the linearly polarized (LP) mode can be applied in the fiber. Based on the coupled-mode theory, the principle of MSC is to reach the phase-matching condition between the fundamental mode \( LP_{\text{in}} \) in the SMF and HOM \( LP_{\text{in}} \) in the MMF. If the effective indices or propagation constants are equal, the phase-matching condition can be satisfied and the power of the \( LP_{\text{in}} \) mode in the MMF can be completely transferred to the \( LP_{\text{in}} \) mode in the MMF with fiber lengths that are odd multiples of the half beat coupling length. For an MSC composed of two dissimilar fibers, the output power in the MMF can be expressed as

\[
P_c = \frac{P_0}{1 + (\kappa \delta)^2} \sin^2 (\sqrt{\kappa^2 + \delta^2} \cdot L),
\]

where \( P_0 \) is the optical power of the HOM in the MMF; \( P_0 \) is the optical power of the fundamental mode in the SMF; \( \delta = (\beta_1 - \beta_2)/2 \) is the phase-mismatch factor; \( \beta_1 \) and \( \beta_2 \) are the propagation constants of the normal mode in the SMF and MMF, which are both equal to \( 2\pi n_{\text{eff}}/\lambda \); \( n_{\text{eff}} \) is the effective index; \( L \) is the coupling length; and \( \kappa \) is the coupling coefficient. The coupling coefficient can be expressed as

\[
\kappa = (-1)^l \frac{2\sqrt{2} k p_1 \Delta n_{\text{in}} \nu_1 \nu_2}{\rho \nu_1 \nu_2} \times \frac{K_0 (\omega_1 d/\rho_1)}{K_0 (\omega_1) K_1 (\omega_2) K_{1,1} (\omega_2)} \cos (\alpha)
\]

where \( u_1 \) and \( u_2 \) are the core modal indices of the SMF and MMF, respectively; \( \omega_1 \) and \( \omega_2 \) are the cladding modal indices of the SMF and MMF, respectively; \( \nu_1 \) and \( \nu_2 \) are the normalized frequencies of the SMF and MMF, respectively; \( l \) is the mode order in the azimuthal direction; \( k \) is the wave number; \( \rho_1 \) and \( \rho_2 \) are the core radii of the SMF and MMF, respectively; \( \Delta n_{\text{in}} \) is the relative index difference in the MMF; \( n_{\text{in}} \) is the fiber-core refractive index; \( d \) is the distance between the SMF and MMF for MSCs; \( K \) is the modified Bessel function of the second kind; and \( \alpha \) is the azimuthal angle shown in Fig. 1 in Ref.32.

It is assumed that the indices of the core and cladding are \( n_{\text{in}} = 1.4392 \) and \( n_{\text{cl}} = 1.431 \) for both the SMF and MMF, respectively; the distance between the SMF and MMF for MSCs is 18 μm; and the core radius of MMF is 9 μm. In order to completely transfer the power from the \( LP_{\text{in}} \) mode in the SMF to the \( LP_{\text{in}} \) mode in the MMF, it is essential that the modes have the same \( n_{\text{in}} \) or \( \beta \). To achieve this, the SMF can be tapered to satisfy phase matching while the MMF has a constant diameter of 18 μm for easy cascading, as in Ref.37. In Fig. 1, the \( n_{\text{in}} \) of the \( LP_{\text{in}} \) mode in the SMF as a function of the core radius (black curve) and those of the \( LP_{\text{in}} \) modes in the MMF (horizontal color line) at a wavelength of \( \lambda = 1550 \) nm are mapped. For realizing phase matching between the \( LP_{\text{in}} \) mode and the \( LP_{1}, LP_{2}, \) and \( LP_{3} \) modes, SMFs with different radii can be employed. Further, the radii of the SMFs of the MSCs (MSC1, MSC2, and MSC3) are equal to 5 μm, 3.21 μm, and 1.98 μm, respectively, as shown in Fig. 1.

![Fig. 1 | Effective indices of different modes in the SMF (LP_{11} mode) and MMF (LP_{21} modes) at a wavelength of 1550 nm.](image)

The black curve shows the effective index of the fundamental mode in the SMF as a function of the core radius. The horizontal lines show the effective indices of the \( LP_{\text{in}} \) modes in the MMF.

Based on the given parameters, we simulate the performance of MSCs. From the analysis in Ref.32, the coupling coefficient between the \( LP_{\text{in}} \) mode and \( LP_{\text{in}} \) mode (i.e., \( LP_{\text{in}} = F_\text{in}(r)\sin(\theta) \), where \( F_\text{in}(r) \) is the radial wave function for \( LP_{\text{in}} \) modes) is zero when the two cores of the MSC are in the same horizontal plane. Thus, if the input mode is the fundamental mode \( LP_{11} \) in the SMF,
only the $LP_{01}$ mode (i.e., $LP_{01} = F_{01}(r)\cos(\theta \rho)$) can be selectively excited in the MMF. Figures 2(a)–2(c) show the normalized power conversion from the $LP_{01}$ mode to the $LP_{01}^c$ mode in MSC1, from the $LP_{01}$ mode to the $LP_{01}^t$ mode in MSC2, and from the $LP_{01}$ mode to the $LP_{01}^{ct}$ mode in MSC3, respectively. As the coupling length increases, the coupling power follows a sinusoidal oscillation. The coupling length $L$ can be designed specifically to realize a maximum coupling efficiency of close to 100% for exciting the $LP_{01}^c$, $LP_{01}^t$, and $LP_{01}^{ct}$ modes in the MMF, and they are equal to 9.8 mm, 8.8 mm and 6.2 mm, respectively. The insets show the simulated intensity profiles of the $LP_{01}^c$ and $LP_{01}^t$ modes.

**Inner elliptical cladding fiber**

Another critical part of the mode converter is the IECF, as shown in Fig. 3. Figure 3(a) shows the cross section and refractive-index distribution of the IECF. The proposed fiber has two claddings, i.e., the inner elliptical cladding and outer circular-symmetry cladding. The elliptical cladding breaks circular symmetry, introduces birefringence, and removes the degeneracy of the fiber modes. The refractive indices of the outer cladding and inner core in the IECF are the same as those of the fiber modes. The refractive-index distribution of the IECF, and Fig. 3(b) shows the simulated intensity profiles of the $LP_{01}$ mode to the $LP_{01}^c$ mode in MSC1 between the SMF and MMF as a function of coupling length $L$. Inset: the elliptical cladding breaks circular symmetry, introduces birefringence, and removes the degeneracy of the fiber modes.

**Fig. 2 | Normalized power conversion**

(a) from the $LP_{01}$ mode to the $LP_{01}^c$ mode in MSC1, (b) from the $LP_{01}$ mode to the $LP_{01}^t$ mode in MSC2, and (c) from the $LP_{01}$ mode to the $LP_{01}^{ct}$ mode in MSC3 between the SMF and MMF as a function of coupling length $L$. Insets: simulated intensity profiles of the $LP_{01}^c$ and $LP_{01}^t$ modes.

Based on the above analysis, the $LP_{01}$ mode can be converted into the $LP_{01}^c$ mode by using MSCs. Next, the generation of OAM modes will be discussed. The $LP_{01}^c$ mode generated by MSCs can be decomposed into two orthogonal degenerate modes $LP_{01}^c$ (even) and $LP_{01}^t$ (odd). By combining the $LP_{01}^c$ and $LP_{01}^t$ modes having a phase difference of $\pm \pi/2$, OAM modes can be generated and expressed as follows:\(^{14,17}\):

$$OAM_{\pm l} = LP_{01}^e \pm r \times LP_{01}^o = F_{01}(r) \exp(\pm i l \theta \rho).$$ \(3\)

If the phase difference is 0 or $\pi$, the combined mode fields can be expressed as follows:\(^{14}\):

$$LP_{01}^e + LP_{01}^o = F_{01}(r) / \sqrt{2} \cos(l \theta \rho), \quad$$

or

$$LP_{01}^e - LP_{01}^o = F_{01}(r) / \sqrt{2} \sin(l \theta \rho),$$ \(4\)

which represents a $45^\circ$ rotation of the $LP_{01}^e$ or $LP_{01}^o$ mode.

When the $LP_{01}^c$ mode passes through the IECF, $n_{eff}$ will be different between the two orthogonal modes because of the birefringence effect. The difference can be expressed as

$$\Delta n_{eff} = n_{eff}(LP_{01}^e) - n_{eff}(LP_{01}^o).$$ \(5\)

In order to design the proper IECF length ($L_{IECF}$) for generating different OAM modes, we calculate the $\Delta n_{eff}$ using COMSOL software and the corresponding beat length ($A$) at a wavelength of 1550 nm, as listed in Table 1, where $A$ can be expressed as $A = \lambda / \Delta n_{eff}$.

**Mode converter**

The evolution diagrams of intensity and phase from the $LP_{01}$ mode to $LP_{01}^c$ modes that are decomposed into two degenerate orthogonal modes $LP_{01}^c$ and $LP_{01}^t$ when the $LP_{01}$ mode passes through the MSCs, and those from two degenerate modes to OAM $3_{11}$, OAM $3_{21}$, and OAM $3_{31}$ modes when $LP_{01}^c$ modes pass through the IECFs are illustrated in Fig. 4. When the $L_{IECF}$ is set to a quarter beat
length ($\lambda/4$), a $\pi/2$ phase difference can be obtained between two degenerate modes, and the OAM$_{11}$, OAM$_{21}$, and OAM$_{31}$ modes can be obtained. The IECF should generate not only the positive TCs but also the negative ones. One can change the sign of the TCs by changing the $\Delta n_{eff}$ accordingly, according to the coupling mode theory, the polarization states of the output modes are the same as that of the input LP$_{01}$ mode.$^{12,36}$.

### Discussion

We also simulated the normalized power evolution between the LP$_{11}$ modes and LP$_{21}$ modes along the propagation length in the IECF, which is analyzed by using the beam propagation method, as shown in Fig. 5. The IECF can be used as a mode rotator for the conversion between two degenerate LP modes, which is similar to the method in Ref.$^{37}$. The intersection points of each pair of curves are the corresponding positions of generation of OAM$_{11}$ modes. The propagation lengths are consistent with those listed in Table 1. By appropriately designing the $L_{IECF}$ and controlling the rotation angle $\theta$, one can selectively generate different OAM modes.

Furthermore, the evolutions with different $L_{IECF}$ values are analyzed. The phase difference between odd and even modes varies with $L_{IECF}$, and the generated optical field can be described as

$$LP_n^p + \exp(i\alpha) \cdot LP_n^{-p} = A_n(r, p, \varphi) \exp[i\alpha(p, \varphi)],$$

(6)

where $A_n$ is the amplitude distribution and $\alpha$ is the phase distribution. These distributions can be expressed as follows:

$$A_n(r, p, \varphi) = F_n(r) \sqrt{1 + 2\cos(l\varphi)\sin(lp)} \cos p,$$

$$\alpha(p, \varphi) = \arctan[\frac{\sin p\sin(l\varphi)}{\cos(l\varphi) + \cos p\sin(l\varphi)}],$$

(7)

where $p=2\pi L_{IECF}/\lambda$ is the phase difference between the odd and even modes. The local helicity of the interference can be expressed as:

$$\frac{\partial\tilde{\alpha}(p, \varphi)}{\partial \varphi} = \frac{\sin p}{1 + \sin(2lp) \cos p}.$$

(8)

The average OAM can be continuously varied with respect to $p$. We choose the generation of the first-order OAM mode as an example. Figures 6(a)~6(d) show the

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Table 1 | Structural parameters of IECFs for the generation of OAM modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>OAM$<em>{11}$/OAM$</em>{-11}$</th>
<th>OAM$<em>{21}$/OAM$</em>{-21}$</th>
<th>OAM$<em>{31}$/OAM$</em>{-31}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta n_{eff}$</td>
<td>$0.5 \times 10^{-4}$</td>
<td>$0.8 \times 10^{-4}$</td>
<td>$0.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Lambda$ (mm)</td>
<td>31</td>
<td>19.4</td>
<td>17.22</td>
</tr>
<tr>
<td>$L_{IECF}$ (mm)</td>
<td>7.75/23.25</td>
<td>4.84/14.53</td>
<td>4.3/12.9</td>
</tr>
</tbody>
</table>
intensity, phase, interference, and local helicity distributions of the OAM mode with \( l = \pm 1 \) generated by MSC and IECF, with different values of \( L_{IECF} \), respectively. As shown in the figure, the OAM mode can be generated with different values of \( L_{IECF} \). However, the variations of phase distributions of the generated OAM mode are not uniform, as shown in Fig. 6(b), and the intensities of the OAM mode are weak. We can also observe the spiral arms to determine the TC of the OAM mode in the interference patterns of the generated OAM beam and a reference Gaussian beam, as shown in Fig. 6(c). As shown in Fig. 6(d), the helicity of the generated OAM mode is also not uniform with the change of \( L_{IECF} \), which implies that the generated OAM mode is not pure. To improve the purity, \( \theta \) should be equal to \( \pi/2 \) or \( 3\pi/2 \) if possible.

Finally, the OAM intensity spectrum of the generated OAM mode is quantitatively analyzed. As shown in Fig. 7, \( |C_1|^2 \) represents the energy of the OAM\(_{\pm 1}\) component over the total field, i.e., the OAM intensity spectrum or energy spectrum\(^{39,40} \). To generate a highly pure OAM mode, \( \omega \) should be \( \pi/2 \) or \( 3\pi/2 \) if possible, which implies that \( L_{IECF} \) should be 7.75 mm or 23.25 mm. Further, the OAM intensity spectrum is close to 1, which indicates that the OAM mode-conversion efficiency is close to 100%. If \( L_{IECF} \) is 0 or 15.5 mm, the coupled mode will be the \( LP_{11}^{45} \) or \( LP_{11}^{-45} \) mode, and the mode intensity spectrum is 50%. Similarly, we can also obtain the properties of OAM modes with \( l = \pm 2 \) or \( \pm 3 \), generated by MSC\(_2\) cascaded with IECF\(_2\) or MSC\(_3\) cascaded with IECF\(_3\), respectively, at different values of \( L_{IECF} \). The purities of OAM modes in these two cases are also nearly 100% when \( L_{IECF} \) is simultaneously equal to odd multiples of \( \Lambda/4 \) for the desired mode and even multiples of \( \Lambda/4 \) for unwanted modes.

Although the MSC we used above is dependent on wavelength, it is a broadband mode converter with a certain bandwidth, as shown in Fig. 4 in Ref.\(^{34} \). One can further expand the response bandwidth of the IECF for matching the MSC by replacing the inner elliptical cladding with tunable functional materials\(^{41} \).

In Ref.\(^{34,35} \), the MSC was used to excite lower-order \( LP \) modes, and the phase shift was achieved using a polarization controller. In contrast to the previous work, we firstly introduce the combination of the IECF with the MSC to generate HO-OAM modes with \( l = \pm 1, \pm 2, \pm 3 \). By appropriately designing the mode converter and changing the parameters of the fiber for supporting higher modes, OAM modes of even higher orders (\(|l|>3\)) can

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**Fig. 5** | Normalized power evolution between the \( LP_{11}^{\pm 1} \) modes and \( LP_{21}^{\pm 1} \) modes along the propagation length of IECFs. The intersection points of each pair of curves are the corresponding positions of generation of OAM\(_{\pm 1}\) modes.

**Fig. 6** | Intensity, phase, interference, and local helicity distributions of the OAM mode with TCs of \( l = \pm 1 \) generated by MSC\(_1\) and IECF\(_1\), at different values of \( L_{IECF} \).
also be generated. However, it is worth noting that the coupling efficiency decreases with the increase of mode order.

Conclusion
In this work, we designed and simulated an all-fiber-based mode converter composed of an MSC and an IECF to generate OAM modes with TCs ranging from \( l = \pm 1 \) to \( l = \pm 3 \) at a mode-conversion efficiency of almost 100%. First, the MSC is utilized to convert the fundamental mode \( LP_{10} \) to \( LP_{20} \) modes, which can be decomposed into two degenerate orthogonal modes \( LP_{10}^{d} \) and \( LP_{10}^{a} \). Subsequently, a phase difference of \( \pm \pi/2 \) between two orthogonal modes can be achieved to convert them to OAM modes after a certain propagation length in the IECF, which is an odd multiple of a quarter beat length. Finally, the OAM modes generated at different lengths of the IECF were quantitatively analyzed. In practice, this OAM mode converter is easy to fabricate with existing optical-fiber drawing techniques and will allow us to take advantage of the increase in transmission capacity and spectral efficiency in an all-fiber communication system based on OAM modes.

References


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Competing interests
The authors declare no competing financial interests.